

# FLUID SIMULATIONS FOR ATMOSPHERIC PRESSURE LOW-TEMPERATURE PLASMAS



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# In collaboration with ....



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V. Pasko's team (CSSL, Penn State University, USA)

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# Outline

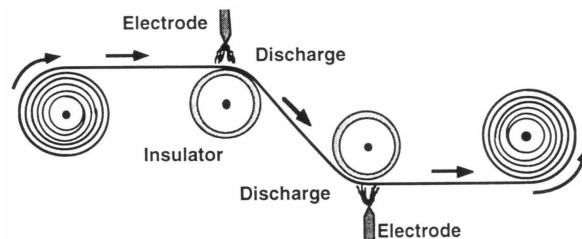
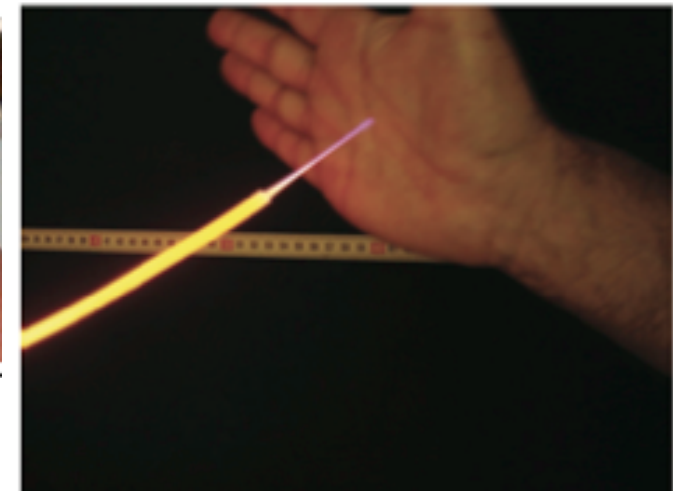
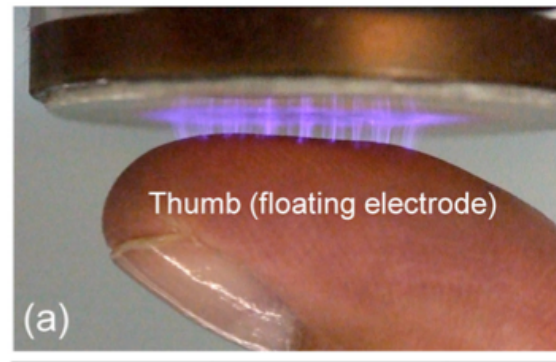
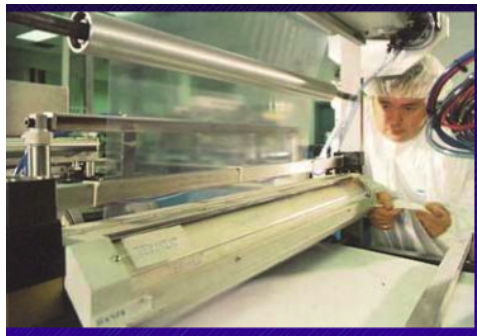


- Introduction on non-thermal discharges at atmospheric pressure
- Rapid overview of the characteristics of streamer discharges
- On the modeling of streamer discharges
- Examples of results
- Challenges in the simulation of non-thermal discharges

# Non-thermal discharges at atmospheric pressure



- Since a few years, many studies on non-thermal discharges at atmospheric ground pressure
  - Wide range of applications at low pressure => possible at ground pressure to reduce costs (no need for pumping systems)?
  - New applications as plasma assisted combustion and biomedical applications

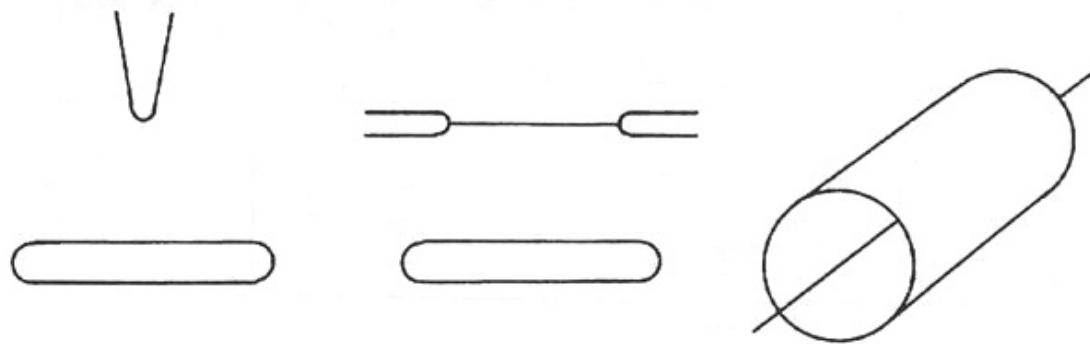




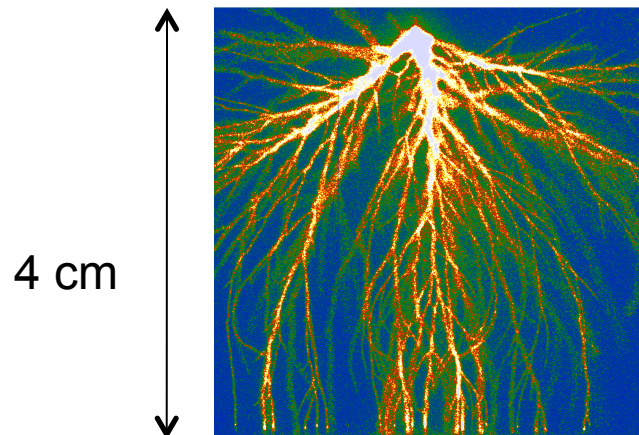
# How to generate non-thermal discharges at atmospheric pressure ?



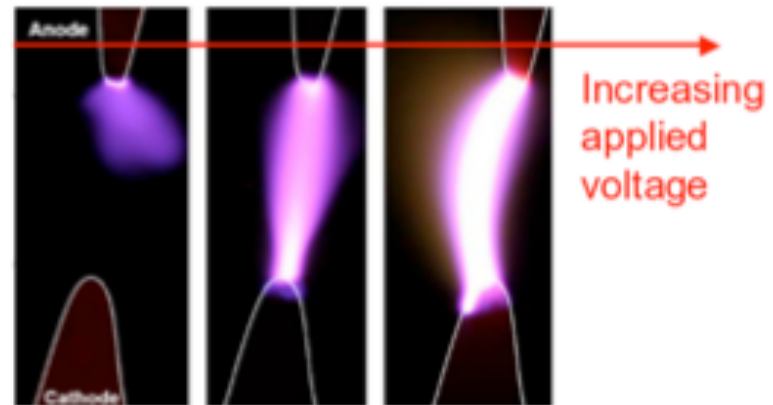
1. Between two metallic electrodes (interelectrode gaps of a few mms to a few centimeters at ground pressure)



Risk : If the voltage pulse is too long => transition to spark



Briels, PhD (2007)



Pai, D., 2008 Ph.D. thesis, Ecole Centrale Paris, France.

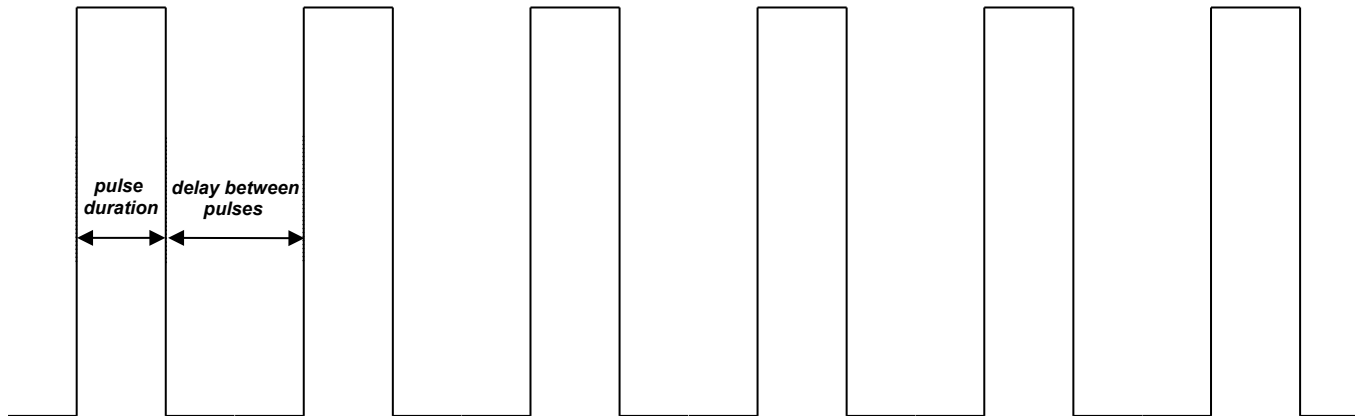
# Nanosecond repetitively pulsed (NRP) discharges in air at Patm (1-30kHz)



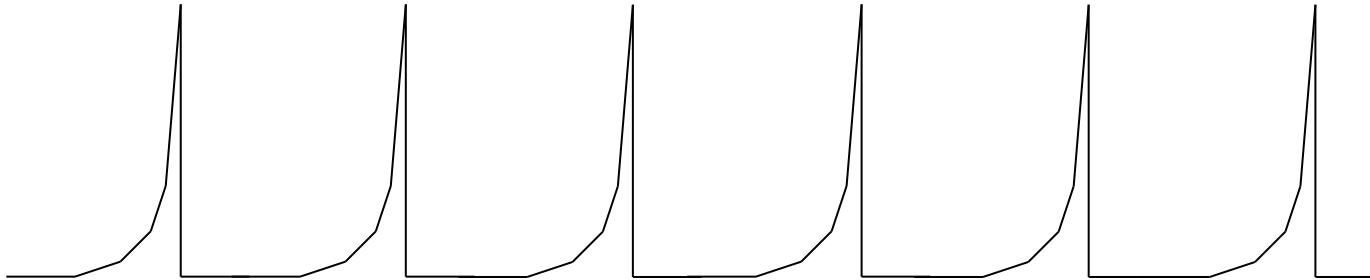
**VOLTAGE:** (field strength  $\sim 100\text{-}300$  Td)

**Pulse duration:**  $10\text{ ns} <$  transition time to spark

**Delay between pulses:**  $33\mu\text{s}\text{-}1\text{ms}$   $\sim$  recombination time



**CURRENT**

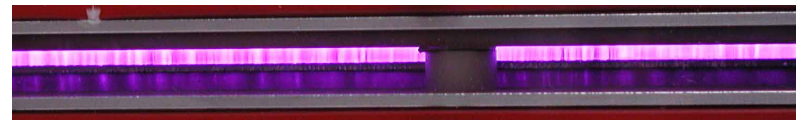
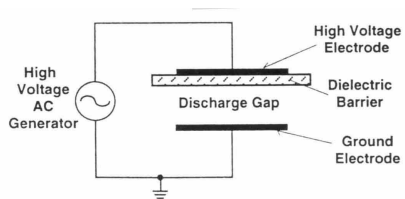


**Energy deposition per pulse is very small  
( $\sim 1$  mJ for  $5\text{ kV} / 10\text{ ns}$  pulses).**

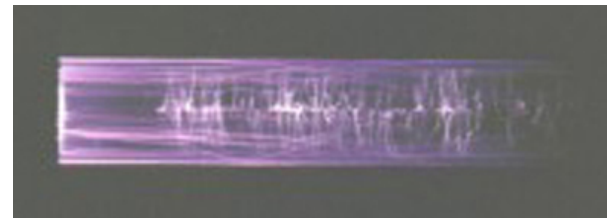
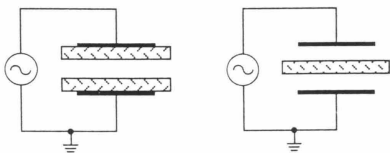
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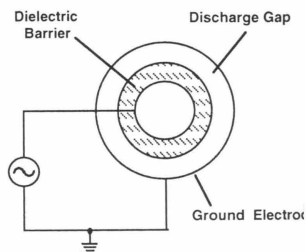
## 2. Dielectric barrier discharges (interelectrode gaps of a few mms to a few centimeters at ground pressure)



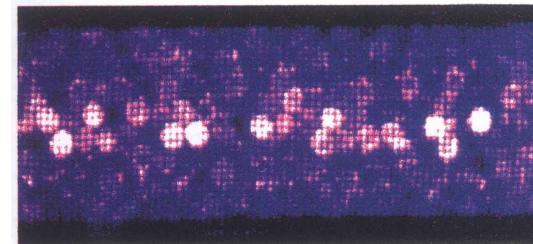
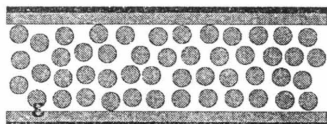
Plane-plane reactor (LPGP Orsay)



Patm discharges



Wire-cylinder (GREMI Orléans)

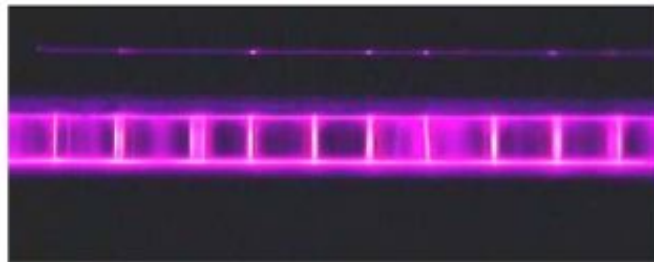


H.Russ et al , *IEEE Trans. Plasma Sci.* **27** (1999) 38

# Non-thermal discharges at atmospheric pressure



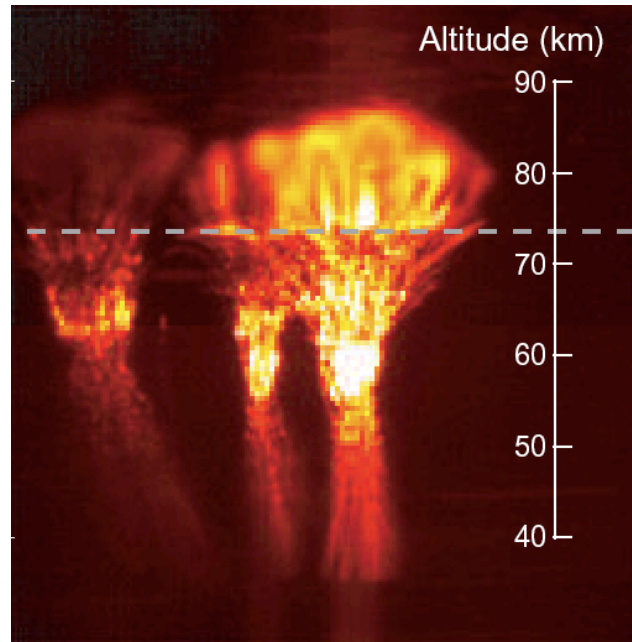
- At Patm, non-thermal plasma discharges are generated in interelectrode gaps of a few mms to a few centimeters  
=> to prevent the transition to the spark regime : short voltage pulses and/or dielectric barrier discharge
- At Patm, non-thermal plasma discharges have filamentary (more frequent) or diffuse structures



**Filamentary discharge (initiated by a streamer discharge):** high concentration of electrons ( $10^{14}\text{cm}^{-3}$ ) in a filament with a radius of the order of  $100\mu\text{m}$   
⇒ high concentrations of active species (radicals, excited species).  
However, local heating may be significant

**Diffuse discharge:** low concentration of electrons, large volume of the discharge and negligible heating

# High altitude discharges



Diffuse and streamer regions of sprites [Stenbaek-Nielsen et al, GRL, 27, 3827, (2000)]

**Similarities of laboratory scale discharges at ground pressure with high altitude discharges:** scaling of air discharge characteristics with pressure (or  $N$ , the gas density) [Pasko et al., GRL, 25, 2123, (1998), Liu and Pasko JPD 39, 327 (2006)]

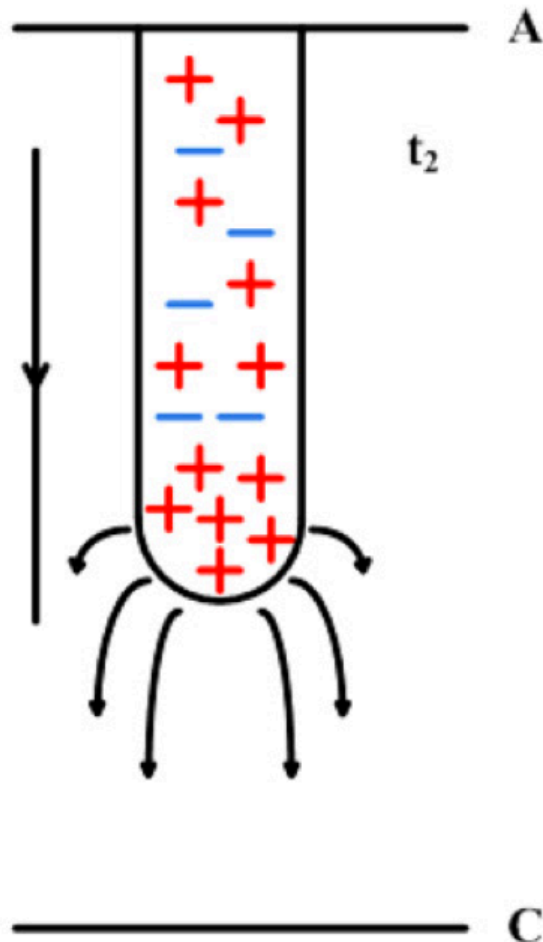
Peak electric field  $\sim N$  Electron density  $\sim N^2$  Time and distance  $\sim N^{-1}$

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- Introduction on non-thermal discharges at atmospheric pressure
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# Positive streamer propagation in air at Patm



## Positive and negative streamers

### Characteristics:

Typical radius of the filament =  $100 \mu\text{m}$

Velocity =  $10^8 \text{ cm/s} \Rightarrow 10 \text{ ns}$  for  $1 \text{ cm}$

Almost neutral channel and charged streamer head

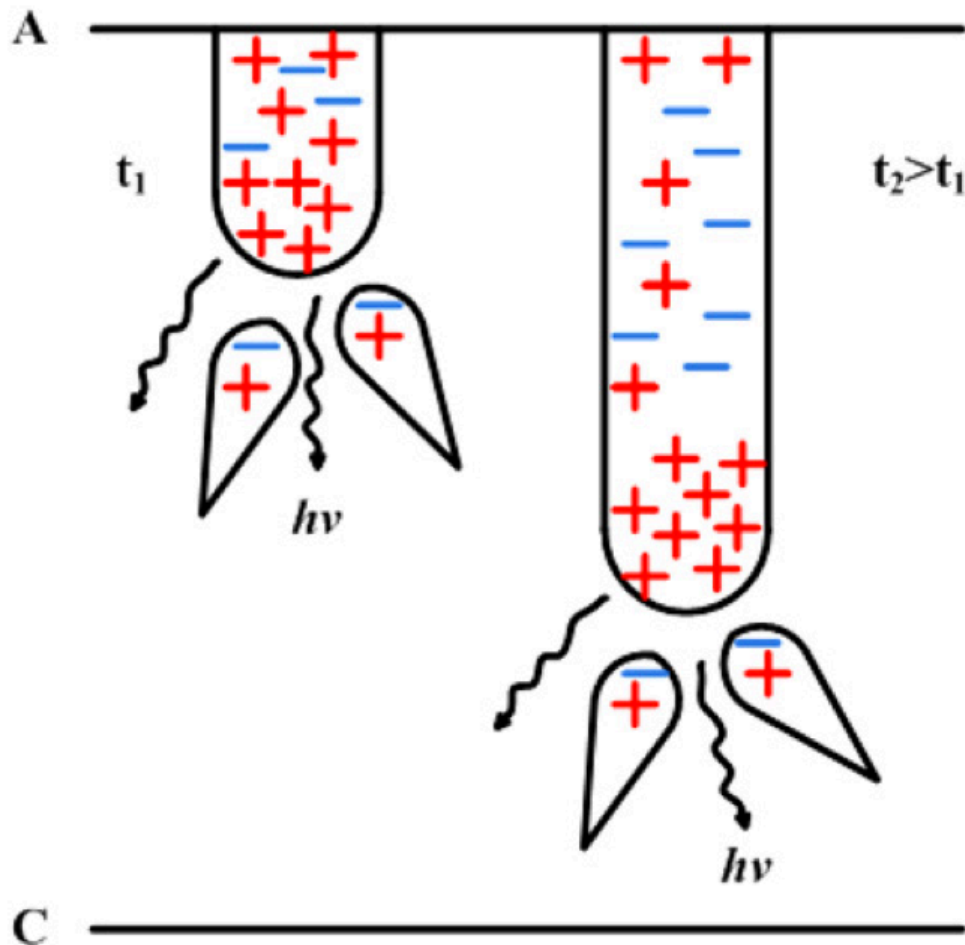
$\Rightarrow$  In the conductive channel: low electric field ( $5 \text{ kV/cm}$ ) and a charged species density of  $10^{13} - 10^{14} \text{ cm}^{-3}$

$\Rightarrow$  In the streamer head: peak electric field ( $140 \text{ kV/cm}$ )

$\Rightarrow$  Plasma frequency  $10 - 100 \text{ GHz}$

$\Rightarrow$  Debye length:  $0.5 - 1 \mu\text{m}$

# Positive streamer propagation in air at $P_{atm}$



Positive streamer propagates from the anode to the cathode  
Drift of electrons in the opposite direction

Ions are almost immobile during propagation

Streamer velocity  $>$  drift velocity of electrons

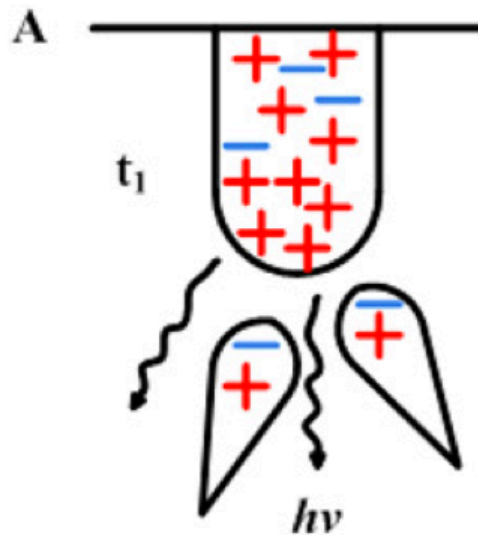
$\Rightarrow$  A streamer discharge is an ionization wave



# Positive streamer propagation in air at Patm

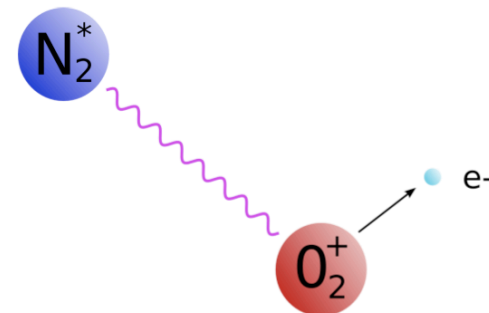


## Need of seed charges for propagation



- Cosmic rays (up to  $10^4 \text{ cm}^{-3}$ ) => too low for streamer propagation

- Photoionization (depends on the gas mixture)  
In air



Ionizing radiation is in the region  
 $980 < \lambda < 1025 \text{ \AA}$ .



- Preionization from previous discharges: At a frequency of 1Hz, preionization level of positive and negative ions of  $10^6$ - $10^7 \text{ cm}^{-3}$  [Wormeester et al. JPD, 43, 505201(2010)]

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# How to simulate non-thermal discharges at Patm?



## Complex medium:

Charged species (ions, electrons), atoms and molecules (excited or not) and photons

⇒ Simplest models take into account only charged species (and photons)

Magnetic effects are negligible:

⇒ Electric field derived from Poisson's equation

## Different models:

⇒ Microscopic model for charged particles coupled to Poisson's equation (PIC-MCC model (Chanrion and Neubert JCP (2008) and JGR (2010))

⇒ Most popular: macroscopic fluid model coupled to Poisson's equation

⇒ Hybrid models:

- Particle model in the high field region ahead of the streamer

- Fluid model in the streamer channel (low field, high electron densities)

(spatially hybrid model for negative streamer (Li, Ebert and Brook IEEE Trans. Plasma Sci. (2008), Li, Ebert, Hundsdorfer, JCP (2012), « bulk-model »

Bonaventura et al., ERL (2014))

# Fluid model for a non-thermal discharge in air at Patm



## Simplest fluid model in air at atmospheric pressure

- Continuity equation is solved for electrons, positive and negative ions

$$\frac{\partial n_i}{\partial t} + \text{div } \mathbf{j}_i = S_i$$

- Poisson's equation

$$\epsilon_0 \nabla \cdot (\epsilon_r \nabla V) = -q_e (n_p - n_n - n_e)$$

$$\mathbf{E} = -\nabla V$$

## Higher order model:

continuity equations, electron energy equation and Poisson's equation

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- ⇒ Strong non-linear coupling between continuity and Poisson's equations
- ⇒ The species densities have to be calculated accurately as their difference is used to calculate the potential and then the electric field
- ⇒ Most models are 2D axisymmetric, few 3D models

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## 2D Fluid model for a discharge in air at Patm

- Continuity equation is solved for electrons, positive and negative ions

$$\frac{\partial n_i}{\partial t} + \text{div}(\mathbf{j}_i) = S_i \quad (1)$$

- Drift-diffusion approximation

$$\mathbf{j}_i = \mu_i n_i \mathbf{E} - D_i \text{grad } n_i \quad (2)$$

- Source terms for air:

$$\begin{cases} S_e = (\partial_t n_e)_{\text{chem}} = (\nu_\alpha - \nu_\eta - \beta_{ep} n_p) n_e + \nu_{\text{det}} n_n + S_{ph} , \\ S_n = (\partial_t n_n)_{\text{chem}} = -(\nu_{\text{det}} + \beta_{np} n_p) n_n + \nu_\eta n_e , \\ S_p = (\partial_t n_p)_{\text{chem}} = -(\beta_{ep} n_e + \beta_{np} n_n) n_p + \nu_\alpha n_e + S_{ph} . \end{cases} \quad (3)$$

- Local field approximation:  $\nu_\alpha(|\vec{E}|/N)$ ,  $\nu_\eta(|\vec{E}|/N)$ ,  $\mu_i(|\vec{E}|/N)$ ,  $D_i(|\vec{E}|/N)$   
Morrow et al., *J.Phys. D:Appl. Phys.* **30**,(1997)

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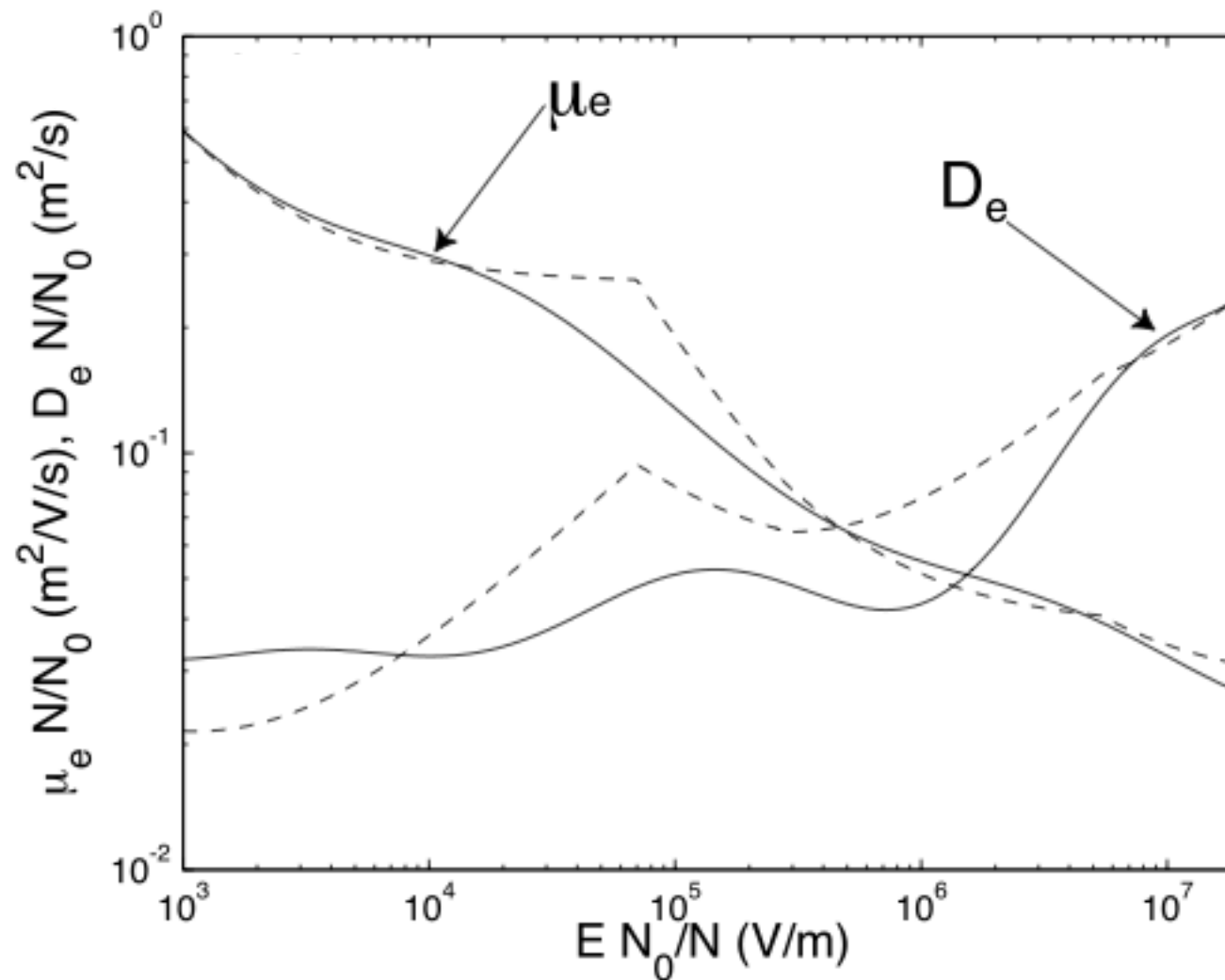
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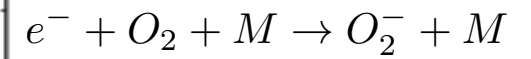
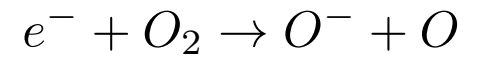
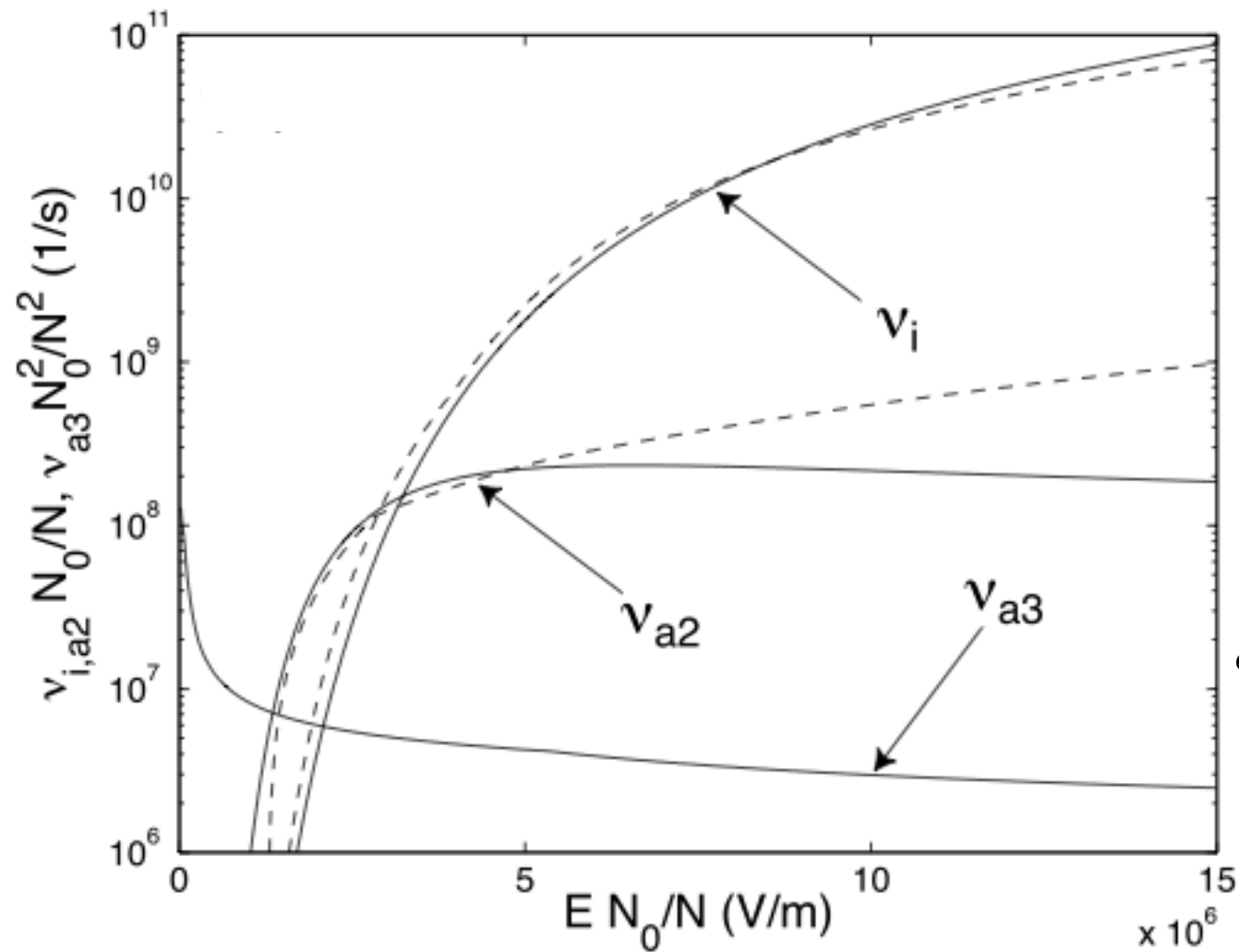
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# Transport coefficients in air at Patm



# Source terms in air at Patm



## 2D Fluid model for a discharge in air at Patm

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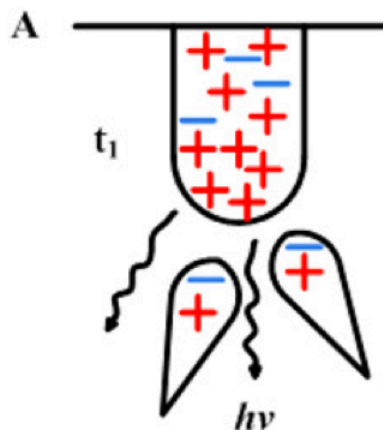
⇒ Need to model the photoionization source term

# Photoionization source term in air



Non-local phenomenon

Photoionization rate at one position depends on all the emitters' positions



Reference model derived from experimental results [Zheleznyak, et al., High Temp., 20, 357 (1982)] and confirmed by recent experiments [Aints, et al., Plasma Process. and Polym. 5, 672 (2008)]

Original model requires to calculate a 3D integral for each point at each time step  
⇒ New model based on a **approximate model for radiative transfer (3 group SP3 model)**

⇒ differential model [Bourdon et al. PSST, 16, 656 (2007), Liu et al. APL 91, 211501 (2007)]



# 2D Fluid model for a discharge in air at Patm



- Poisson's equation with surface charges

$$\epsilon_0 \nabla \cdot (\epsilon_r \nabla V) = -q_e (n_p - n_n - n_e) \quad (4)$$

**BC: Surface charges**  $\sigma$  on dielectric surfaces

- Photoionization source term  $S_{ph}$ : SP3 model  
Bourdon et al., *Plasma Sources Sci. Technol.* **16**, (2007)
- On the dielectric plane surfaces: secondary emission due to ions bombardment  
 $\gamma = 0.1$  (high value to compensate the other emission processes)

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$$\begin{cases} \nabla^2 \phi_{1,j}(\vec{r}) - A_{1,j} \phi_{1,j} = S_{1,j} \\ \nabla^2 \phi_{2,j}(\vec{r}) - A_{2,j} \phi_{2,j} = S_{2,j}; \end{cases} \quad (5)$$

$\lambda_{j=1,3} \rightarrow 2$  Poisson's equation ( $\phi_{1,j}$  and  $\phi_{2,j}$ )  $\times 3$  iterations for BC

$$\boxed{6 \times 3 \text{ Poisson's equations}} \xrightarrow{\downarrow} S_{ph} = \sum_j = f(\phi_{1,j}(\vec{r}), \phi_{2,j}(\vec{r}))$$

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# 2D Poisson's equation

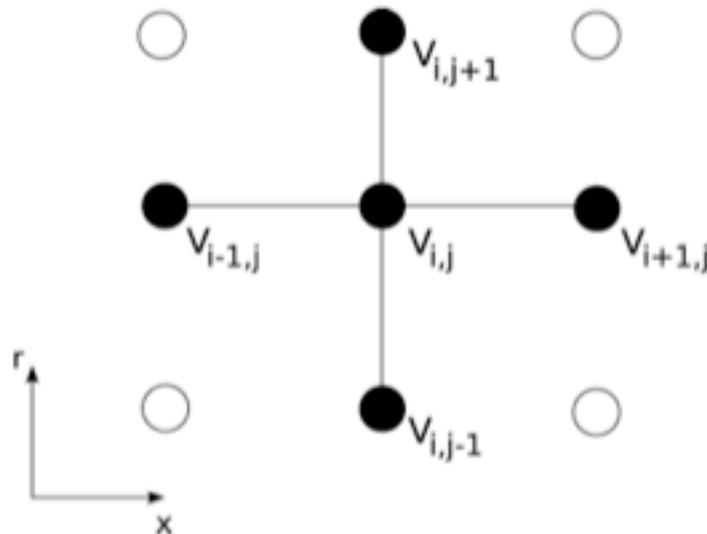


In cylindrical coordinates, Poisson's equation can be written as:

$$\vec{\nabla}^2 V = \frac{\partial}{\partial x} \left( \frac{\partial V}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) = -\frac{\rho(x, r)}{\epsilon_0} \quad (5)$$

Discretization:

$$V_{i,j}^e V_{i+1,j} + V_{i,j}^w V_{i-1,j} + V_{i,j}^s V_{i,j-1} + V_{i,j}^n V_{i,j+1} + V_{i,j}^c V_{i,j} = -\frac{\rho_{i,j}}{\epsilon_0} \quad (6)$$



# 2D Poisson's equation



Besoin de résoudre l'équation de Poisson à chaque pas dans le temps:

## Différentes approches:

- Algorithme basé sur la transformée de Fourier rapide (Kunhardt 85).
- Méthodes itératives: sur-relaxation (Kulikovsky 96), module de la bibliothèque NAG
- Solveurs directs (superLU, MUMPS, PASTIX)

Les besoins en mémoire des solveurs directs limitent leur utilisation pour des domaines de calculs avec plus de 1 million de points (30 Go pour la version OPENMP de Pastix)

Récemment de nouveaux solveurs itératifs performants (faible besoin en mémoire) :

=> HYPRE library (SMG solver) : hybrid MPI-OPENMP library

# 2D Poisson's equation



## Conditions aux limites:

- Comme l'équation de Poisson est une équation elliptique, un soin particulier doit être porté aux conditions aux limites!
- Attention de bien vérifier l'influence des conditions aux limites (potentiel imposé ou gradient nul) sur les résultats.
- Si la décharge est entre deux électrodes métalliques: potentiel imposé aux électrodes - loin de l'axe de la décharge: s'assurer que le potentiel tend vers zéro.
- Si décharge à barrière diélectrique: prendre en compte le dépôt de charges au cours du temps

# Simulation of streamer discharges



- Streamer discharge simulation are known to be computationally **expensive**
- Temporal multiscale nature of **explicit** streamer simulation:  $\Delta t = 10^{-12} - 10^{-14}$  s

$$\begin{array}{ll} \text{Convection:} & \Delta t_c = \min \left[ \frac{\Delta x_i}{v_{x(i,j)}}, \frac{\Delta r_j}{v_{r(i,j)}} \right] \\ \text{Diffusion:} & \Delta t_d = \min \left[ \frac{(\Delta x_i)^2}{D_{x(i,j)}}, \frac{(\Delta r_j)^2}{D_{r(i,j)}} \right] \\ \text{Chemistry:} & \Delta t_l = \min \left[ \frac{n_{k(i,j)}}{S_{k(i,j)}} \right] \\ \text{Diél. relaxation:} & \Delta t_{Diél} = \min \left[ \frac{\epsilon_0}{q_e \mu_e(i,j) n_e(i,j)} \right] \end{array}$$

- Time scale of streamer propagation in centimeter gaps is  $\sim 10$  ns,  $\rightarrow \sim 10^4$  time steps
- For centimeter gaps of 1 cm,  $\Delta x, r = 10 - 1 \mu\text{m} \rightarrow$  nbre of points  $> 1 \times 10^6$

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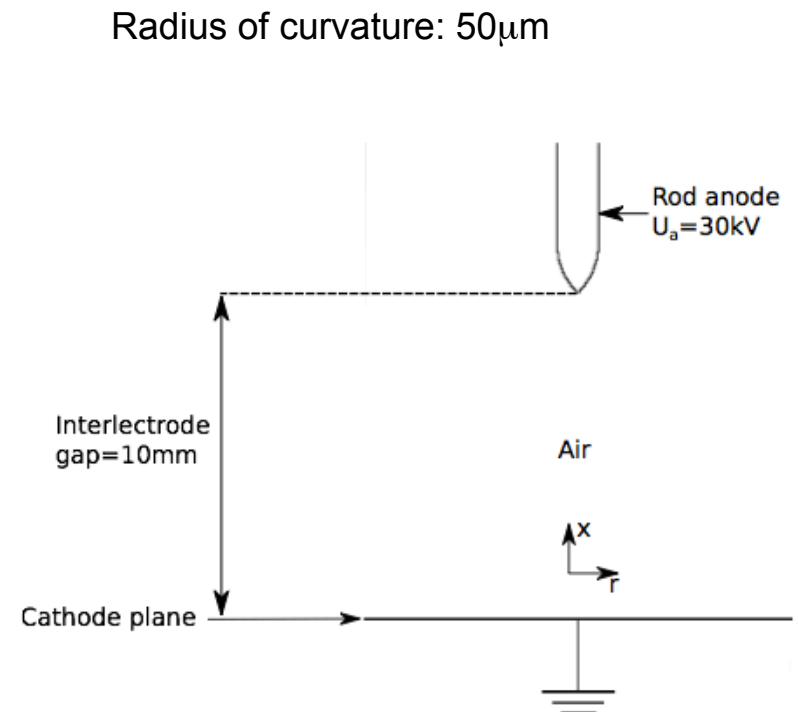
# 2D sequential discharge code

- 2D-axisymmetric discharge code
- Full explicit sequential code using Cartesian non-uniform static mesh
- MUMPS direct solver for Poisson's equation and photo-ionization source term
- Explicit Improved Scharfettel-Gummel (ISG) scheme for the convection-diffusion equation  
Kulikovskiy A., *Journal of Computational Physics* **11**, 149-155,(1995)
- 4<sup>th</sup> order Runge-kutta scheme for the chemistry source term
- 1<sup>st</sup> order operator splitting method:  $U^{t+\Delta t} = CD^{\Delta t} R^{\Delta t} U^t$
- **Verification of the code:**  
Celestin et al, *Journal of Physics D:Applied Physics*. **42**, 065203 (2009)  
S. Celestin, PhD thesis, (2008)
- **Validation of the code:**  
Jánský et al., *Journal of Physics D:Applied Physics*. **44**, 335201 (2011)

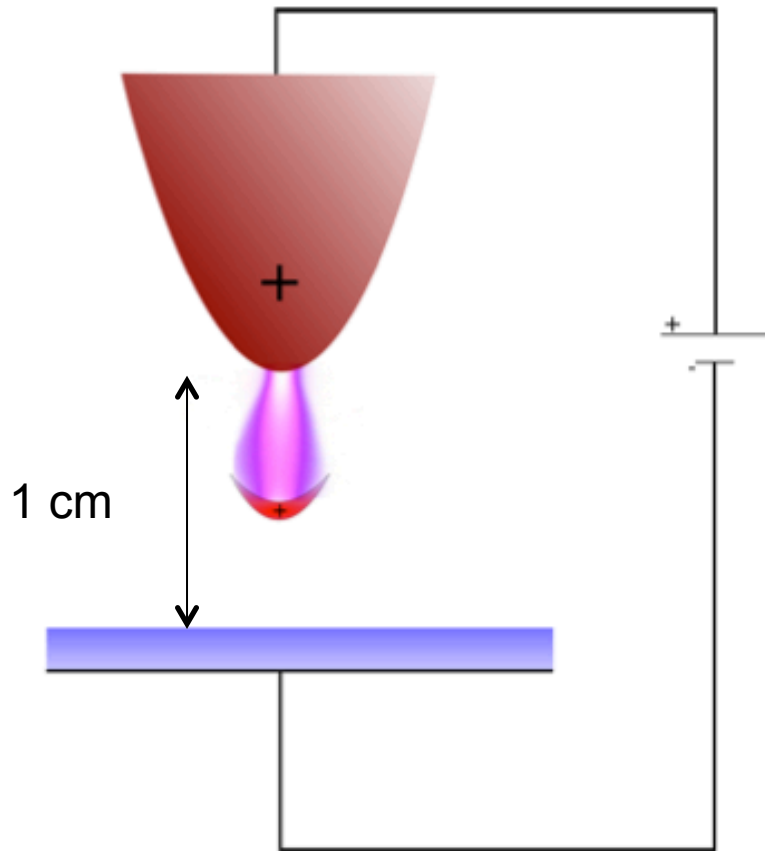
# Test-case: point-to-plane geometry - air at Patm



- Comparison with experiments: **Test case**  
10 mm gap with a sharp point electrode
- Constant voltage applied at the anode,  
 $V_{\text{anode}} = +30 \text{ kV}$  (current is an output)
- Computational domain is  $2 \text{ cm} \times 2 \text{ cm}$  with Cartesian grid
- Large domain size  $n_x \times n_r = 3353 \times 1725$   
so  $5.8 \times 10^6$  points



# Streamer propagation in air at Patm



⇒ In air at atmospheric pressure, the breakdown electric field is 30 kV/cm

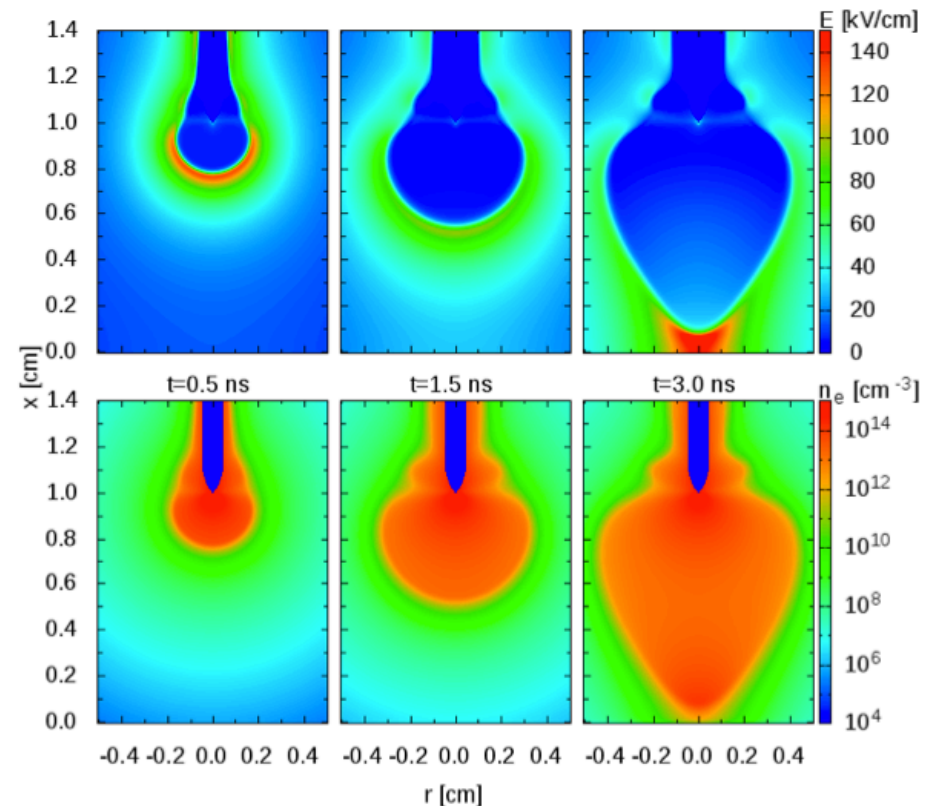
⇒ In a point to plane geometry, the electric field is enhanced close to the point electrode

⇒ A first discharge will start from the point electrode and will propagate towards the grounded plane

# Performance of the discharge code: Test-case



- Ignition of a large positive streamer discharge
- At  $t = 1.5$  ns, maximum diameter of the discharge ( $D = 8$  mm) and  $n_e = 10^{15} \text{ cm}^{-3}$
- At  $t_c = 3.0$  ns, discharge impacts the cathode plane
- Time step:  $\Delta t = \Delta t_{Diel} \sim 10^{-14} \text{ s}$ , dielectric relaxation time step  $\Delta t_{Diel}$  10 times smaller than  $\Delta t_c$ ,  $\Delta t_d$ ,  $\Delta t_l$
- Simulation time :  $\sim$  **one month** with original code (memory used  $> 30$  Go)



[Péchéreau, PhD (2013)]

Mesocenter of Ecole Centrale Paris:

Altix ICE 8400 LX of 68 nodes with two processors six-core Intel Xeon X5650 (2.66Ghz) per node, so 816 cores in total with 24Go of memory per node.

# Performance of the discharge code: Test-case



- One time-step  $\Delta t$ : more than 50 % of the time for solving Poisson's equation
- Potential  $V$  + photoionization source term  $S_{ph}$ :  $1+6 \times 3$  Poisson's equation to solve
- Save computational time:  $S_{ph}$  is computed every 5 time steps (negligible influence on results)
- In original code, direct solver MUMPS to solve Poisson's equation:
  - $1 \times 10^6$  points  $\rightarrow$  Memory (factorization): 520 Mo  $\times (1+6) = 3.7$  Go
  - $6 \times 10^6$  points  $\rightarrow$  Memory (factorization): 4 Go  $\times (1+6) = 28$  Go

## Limitations of the initial discharge code:

- Number of points for large simulated domains
- Solution time to solve Poisson's equation
- Small time-step  $\Delta t = \Delta t_{Diel} \sim 10^{-14}$ s

# Performance of the discharge code: Test-case

## Limitations of the initial discharge code:

- Number of points for large simulated domains
- Solution time to solve Poisson's equation
- Small time-step  $\Delta t = \Delta t_{Diel} \sim 10^{-14}\text{s}$

- **Number of points:** Adaptive Mesh Refinement (AMR)

- ▶ Parallel (MPI) AMR code (use of PARAMESH) with a fluid model for the simulation of filamentary discharge (2D-3D)

Pancheshnyi et al., *Journal of Computational Physics* **227**, (2008)

- ▶ Parallel (MPI) AMR code with a hybrid particle-fluid model for the simulation of streamer discharge (2D-3D)

Kolobov et al., *Journal of Computational Physics* **231**, (2012)

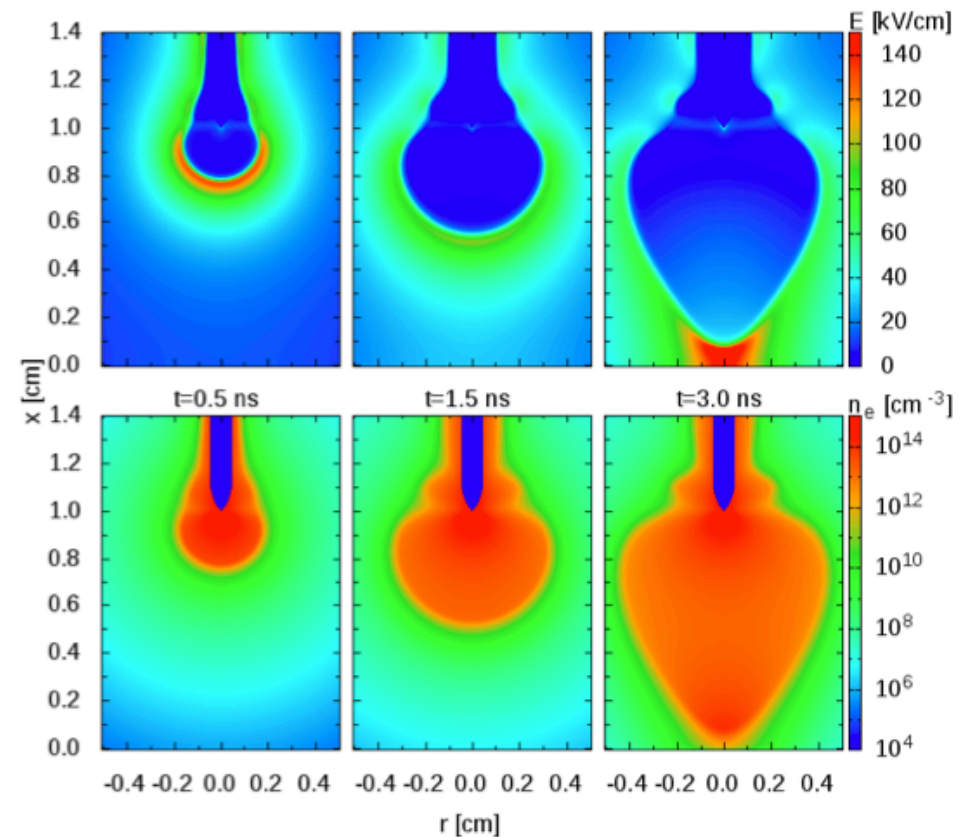
- 1 Small time-step  $\Delta t = \Delta t_{Diel} \sim 10^{-14}\text{s} \Rightarrow$  'semi-implicit' scheme
- 2 Solution time to solve Poisson's equation + memory (nbre points=  $6 \times 10^6$  points)  $\Rightarrow$  for large domains, current iterative solvers are competitive (low memory)



# Improvements of the discharge code: Poisson's solver

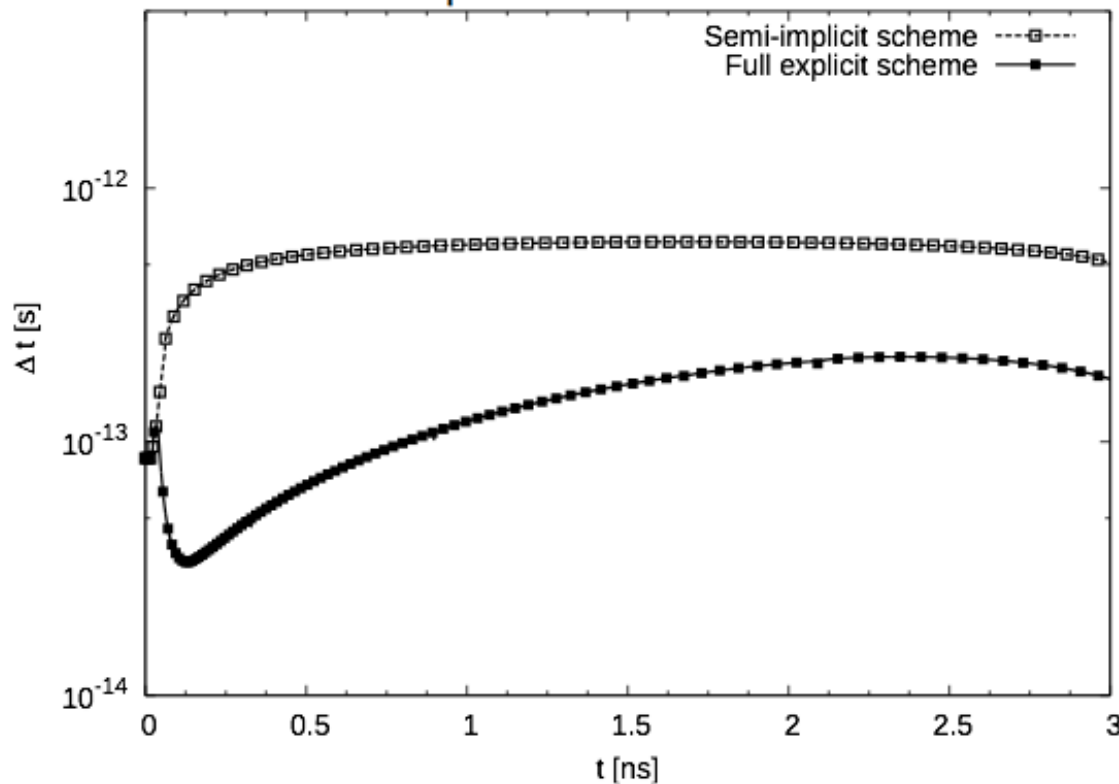


- On the test-case (TC), with the MUMPS direct solver, memory required > 30 Go
- Large simulated domain → iterative solver becomes competitive
- Implementation of the parallel MPI-OPENMP SMG solver (HYPRE library)
- Test on TC of laplacian potential:  
72 MPI processes: 5.7 s ↘ 0.5 s
- Test on TC of laplacian potential:  
24 MPI × 3 OPENMP: 5.7 s ↘ 0.4 s
- For iterative solver SMG, memory required is less than 1 Go
- Solved memory problem and solving time, what about the dielectric relaxation time step  $\Delta t_{Diel}$  constraint ?



# Improvements of the discharge code: "semi-implicit" scheme

- To remove  $\Delta t_{\text{Diel}}$ , implementation of a "semi-implicit" scheme  
Lin et al., *Computer Physics Communications* **183**, (2012)
- On the test-case, we compare the implementation with the "semi-implicit" scheme with the full explicit model:



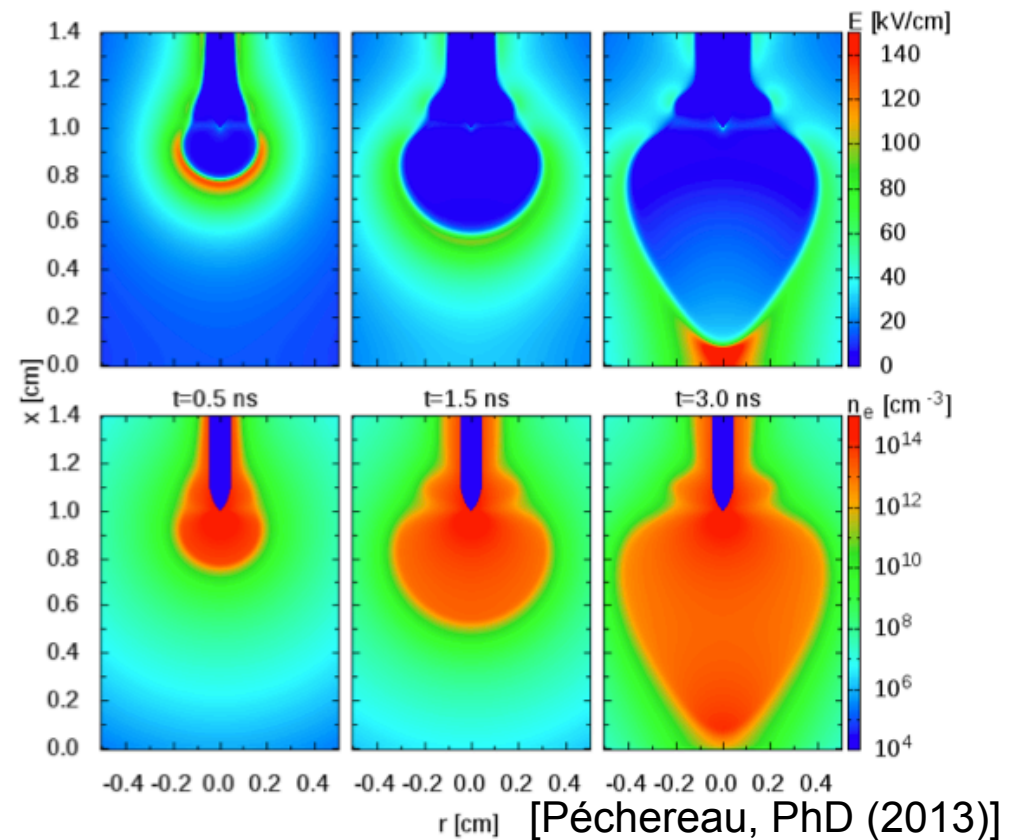
- We can choose a time-step 10 bigger than with the explicit model



# Performance of the discharge code: Test-case



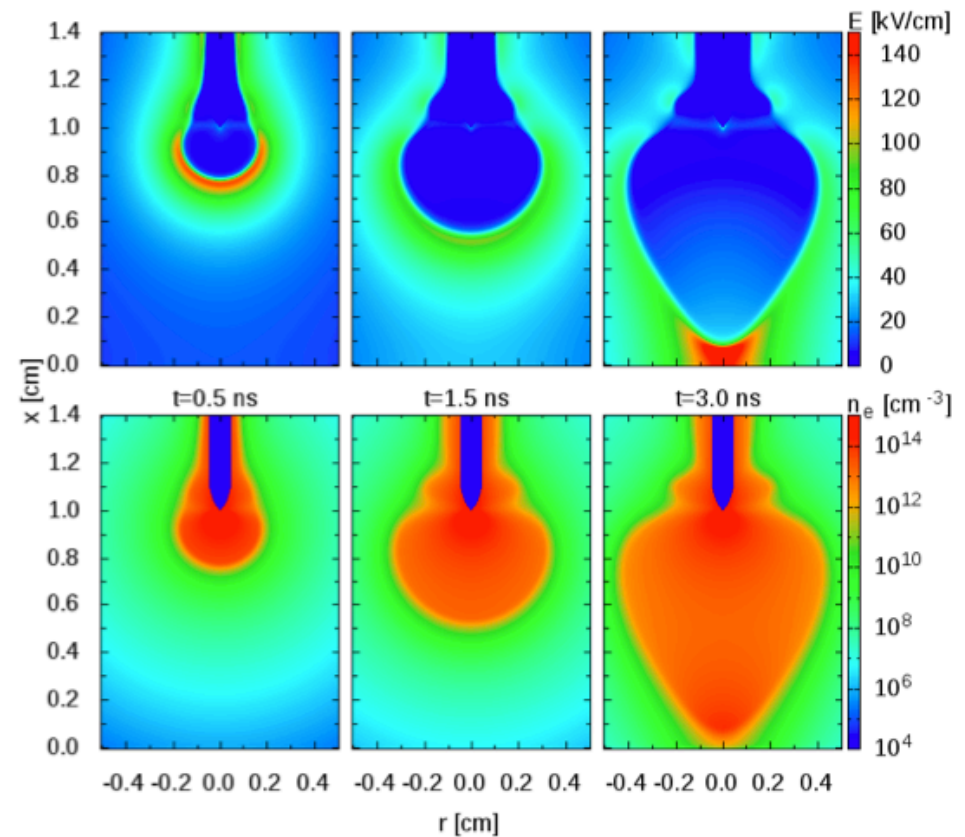
- **Full parallel MPI-OPENMP discharge code with domain decomposition**
- **Poisson's equation:** MPI-OPENMP iterative solver SMG (HYPRE library)
- **Small time-steps:** Semi-implicit scheme (to remove  $\Delta t_{\text{Diel}}$ )
- **Robustness:** Explicit UNO3 scheme 3<sup>rd</sup> order for convection + Explicit 2<sup>nd</sup> order for diffusion (not shown here)
- Test on TC one time-step:
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24 MPI × 3 OPENMP: 17.05 s ↘ 0.86 s
- TC is computed in ~ **3 hours**  
(one month with initial code)



# Performance of the discharge code: Test-case



- **Full parallel MPI-OPENMP discharge code with domain decomposition**
- **Poisson's equation:** MPI-OPENMP iterative solver SMG
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72 MPI processes: 17.05 s ↘ **0.63 s**
- Test on TC one time-step:  
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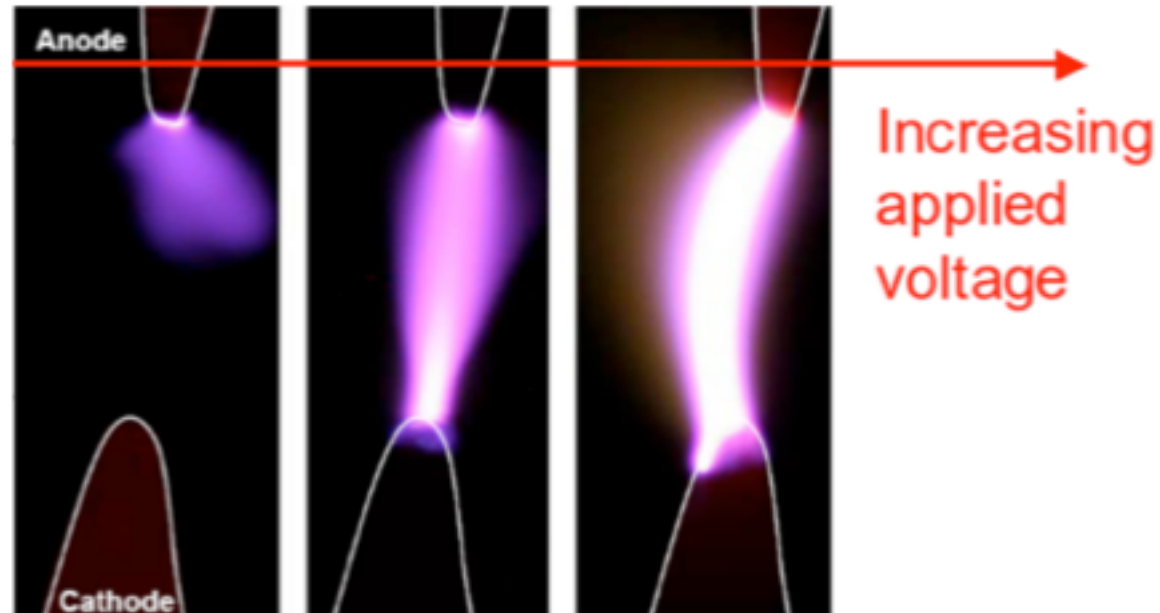


# Outline

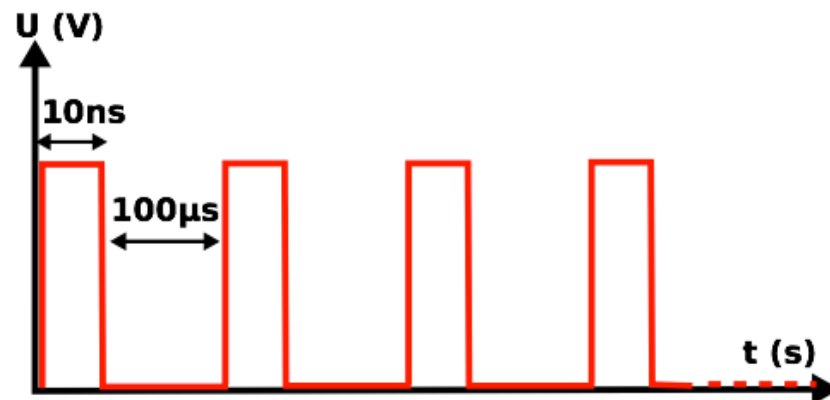


- Introduction on non-thermal discharges at atmospheric pressure
- Rapid overview of the characteristics of streamer discharges
- On the modeling of streamer discharges
- Example of results
  - Dynamics of a nanosecond discharge between point electrodes
  - Interaction of a dielectric barrier discharge with an obstacle
- Challenges in the simulation of non-thermal discharges

# Example of a nanosecond repetitively pulsed discharge in air at atmospheric pressure (NRPD)



Pai, D., 2008 Ph.D. thesis, Ecole Centrale Paris, France.

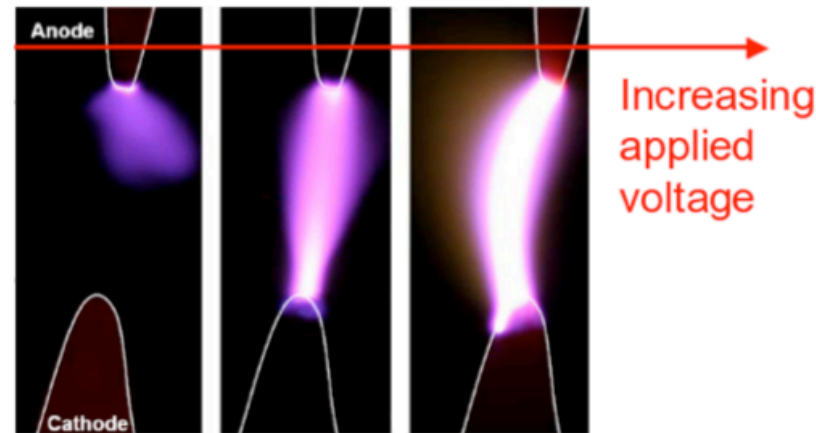


# Nanosecond corona, glow and spark regimes



## Experiments in air:

- $T=1000$  K
- Electrode gap : 2-5 mm
- Frequency: 10-30 kHz
- Pulse duration : 10 ns



Pai, D., 2008 Ph.D. thesis, Ecole Centrale Paris, France.

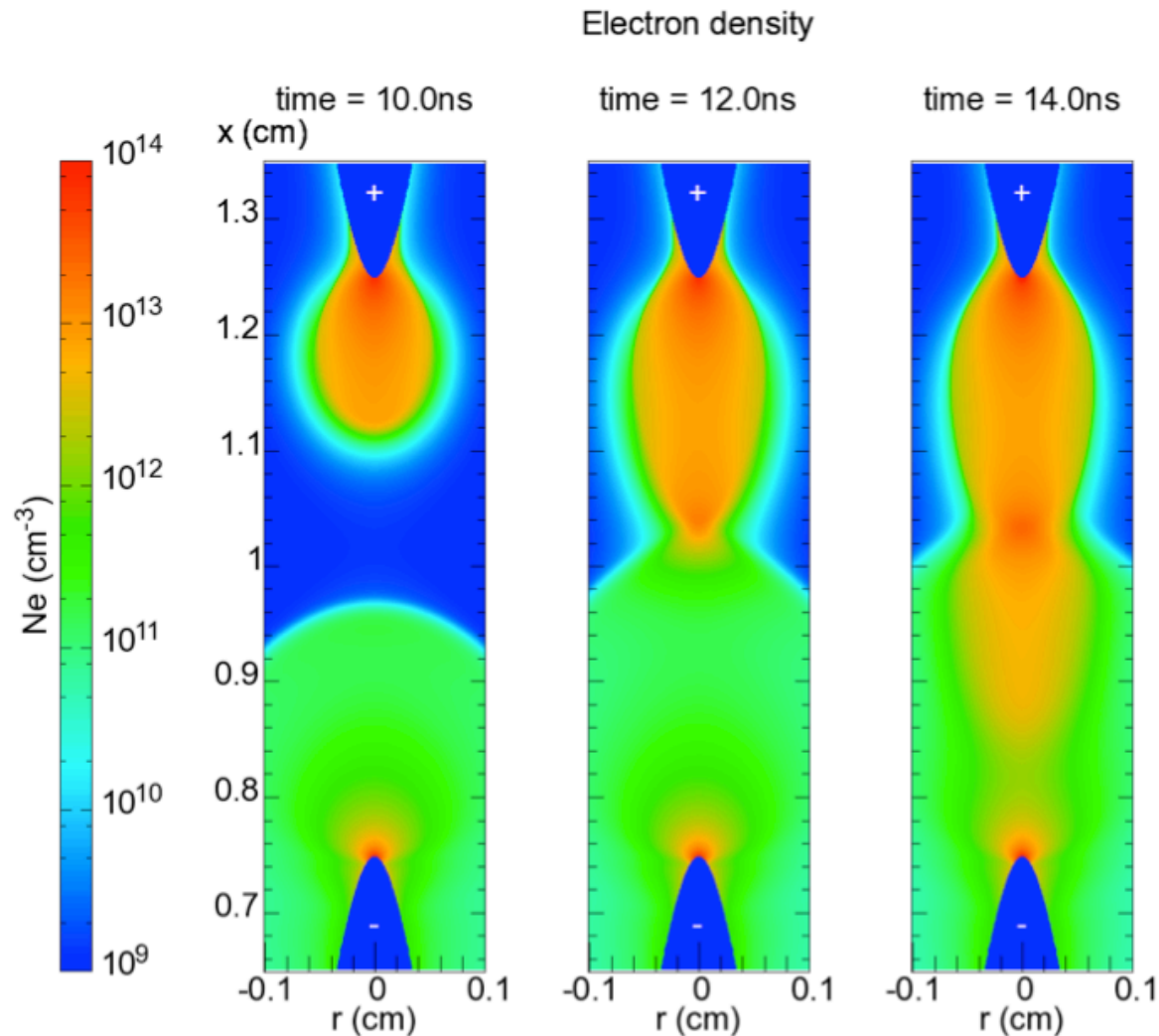
## Simulations of the discharge during one voltage pulse:

- Fluid model for air: 3 species ( $p$ ,  $n$ ,  $e^-$ )
- Drift-diffusion + Poisson equation
- 2D axi-symmetric
- Photoionization
- Voltage pulse : "Sigmoid shape"

# Dynamics of a nanosecond discharge in air at Patm



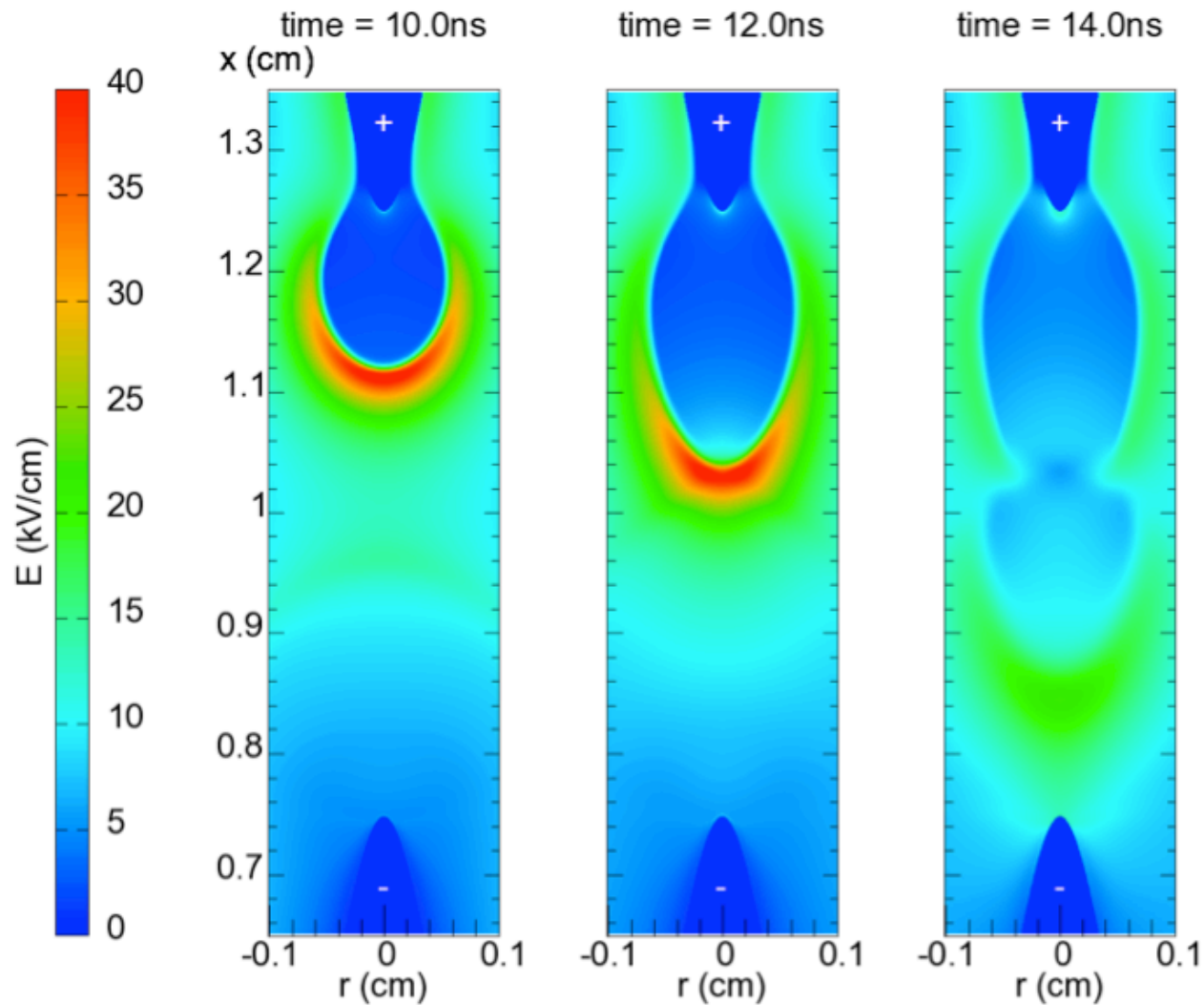
$T=1000$  K,  $V=5$  kV,  $R=50$   $\mu\text{m}$ , gap= 5mm



# Dynamics of a nanosecond discharge in air at $P_{atm}$



$T=1000$  K,  $V=5$  kV,  $R=50$   $\mu\text{m}$ , gap= $5$  mm





# Nanosecond repetitively pulsed discharge (NRPD) in air at atmospheric pressure



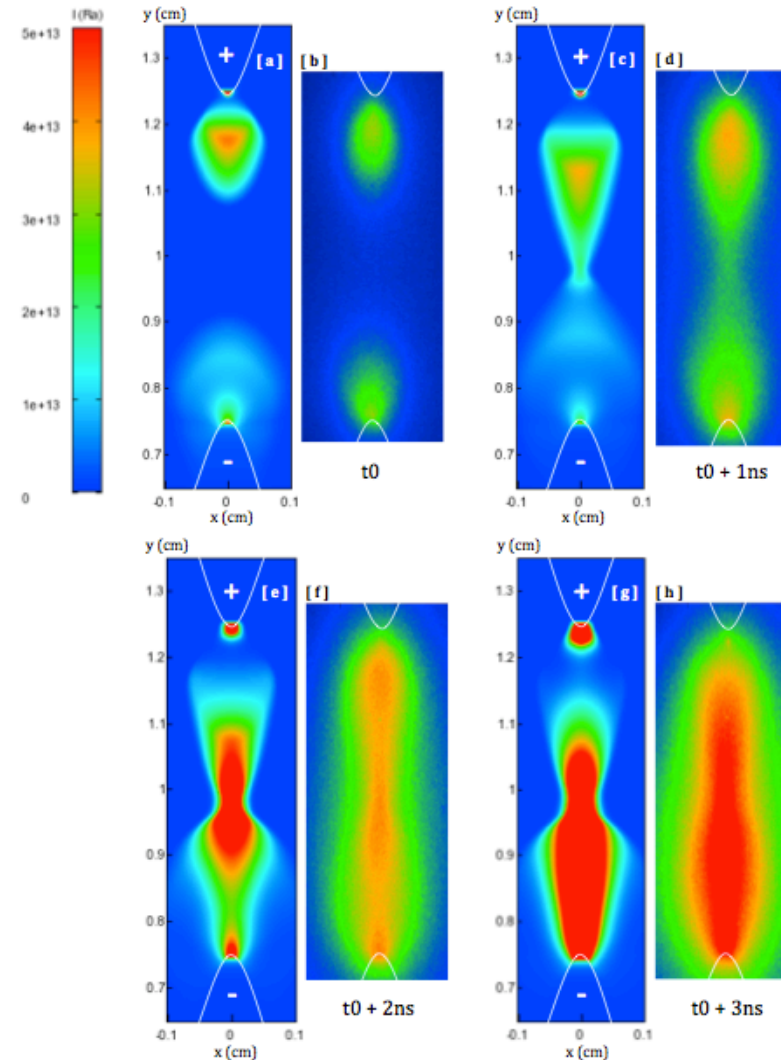
## Comparison experiments/simulations :

### Experimental conditions :

- T=300 K, Flow : 10m/s
- Electrode gap : 5mm
- Frequency: 1kHz
- Pulse duration : 10 ns
- Electrodes : Hyperboloid, R=50 $\mu$ m
- Anode : +9kV, cathode: -9kV
- images :
  - every 1ns
  - 50 accumulations
  - integration time : 2ns

### Simulation conditions :

- T=300 K
- Electrode gap : 5mm
- Preionization :  $10^9 \text{cm}^{-3}$
- Electrodes : Hyperboloid, R=100 $\mu$ m
- Anode : +15kV, cathode : 0V
- images :
  - emission of N<sub>2</sub>(2p)
  - time-integrated over 2ns
  - Abel integrated





# Outline

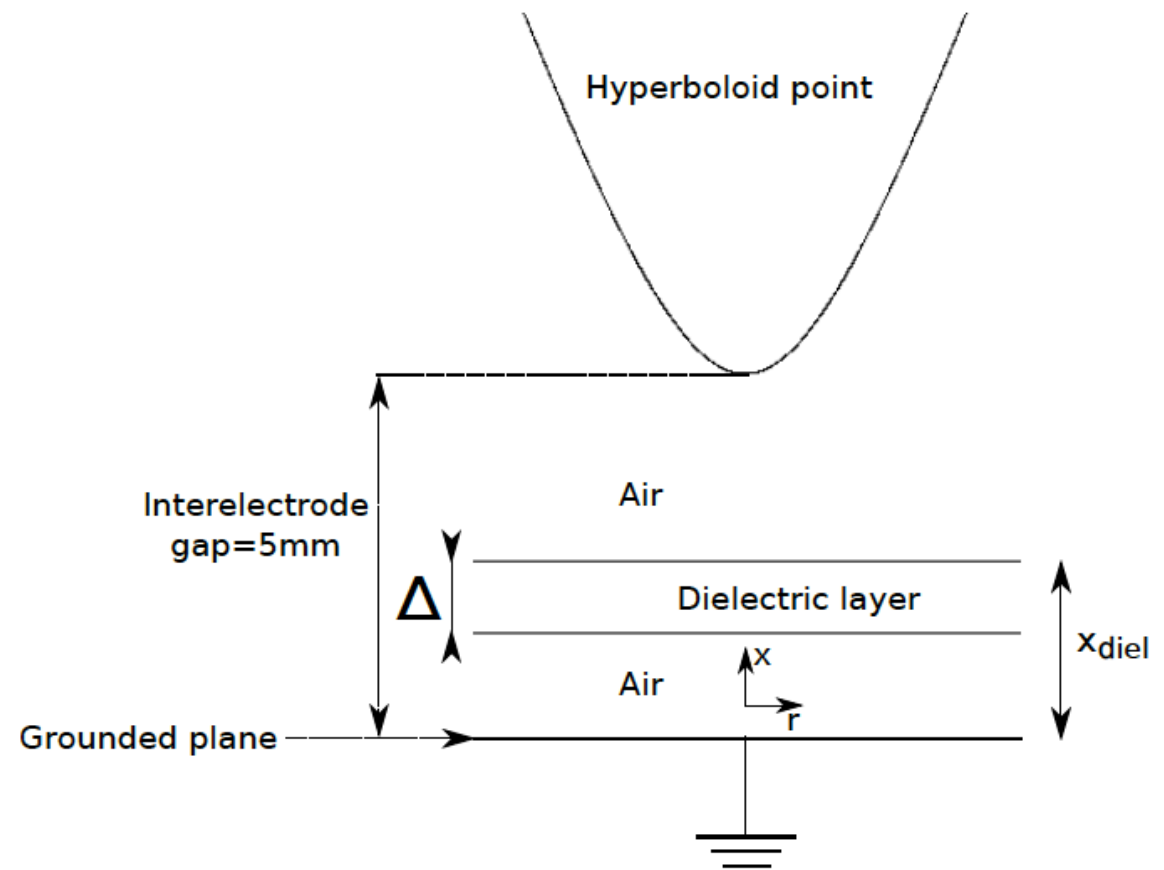


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# Schematic of the discharge



- 2D-axisymmetric code
- Point to plane geometry
- Constant applied voltage at the point electrode
- Dielectric layer placed in the interelectrode gap opaque to radiation
- $\Delta = 176\mu\text{m}$  and  $\epsilon_r = 5$
- Low preionized homogeneous background  
 $n_{\text{init}}^{e,p} = 10^4\text{cm}^{-3}$

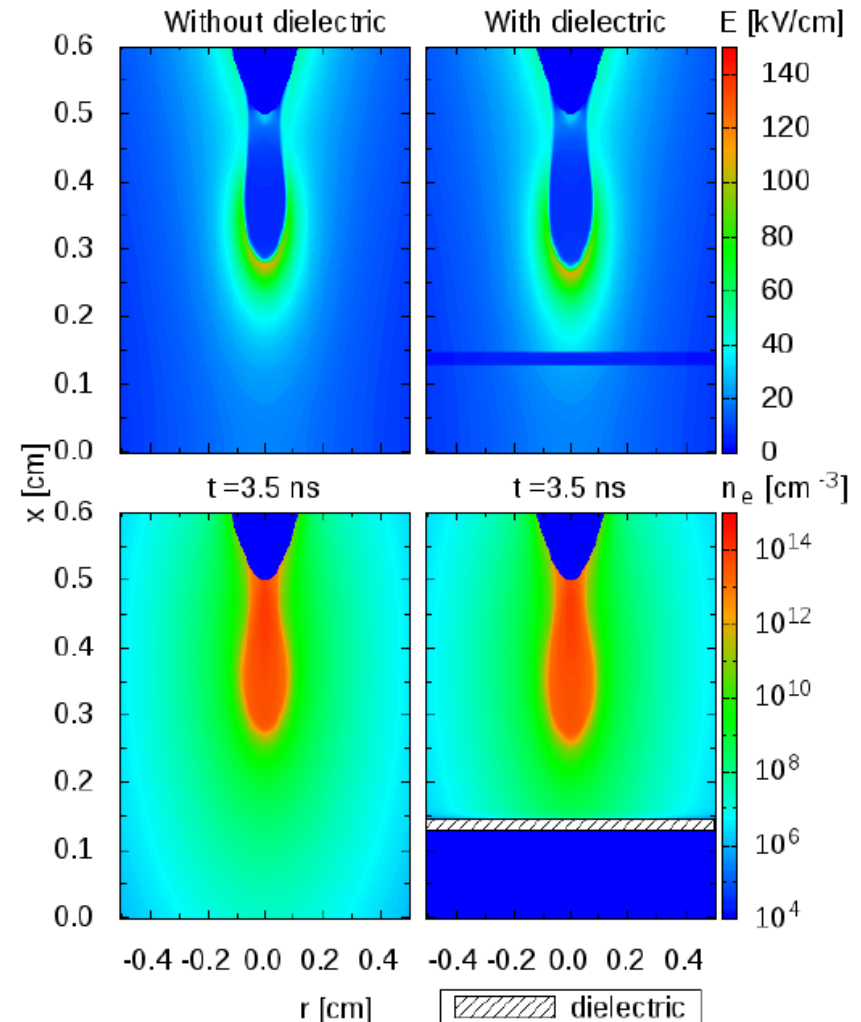


# Influence of a dielectric plane



Constant applied voltage  $V_{\text{anode}} = +13 \text{ kV}$

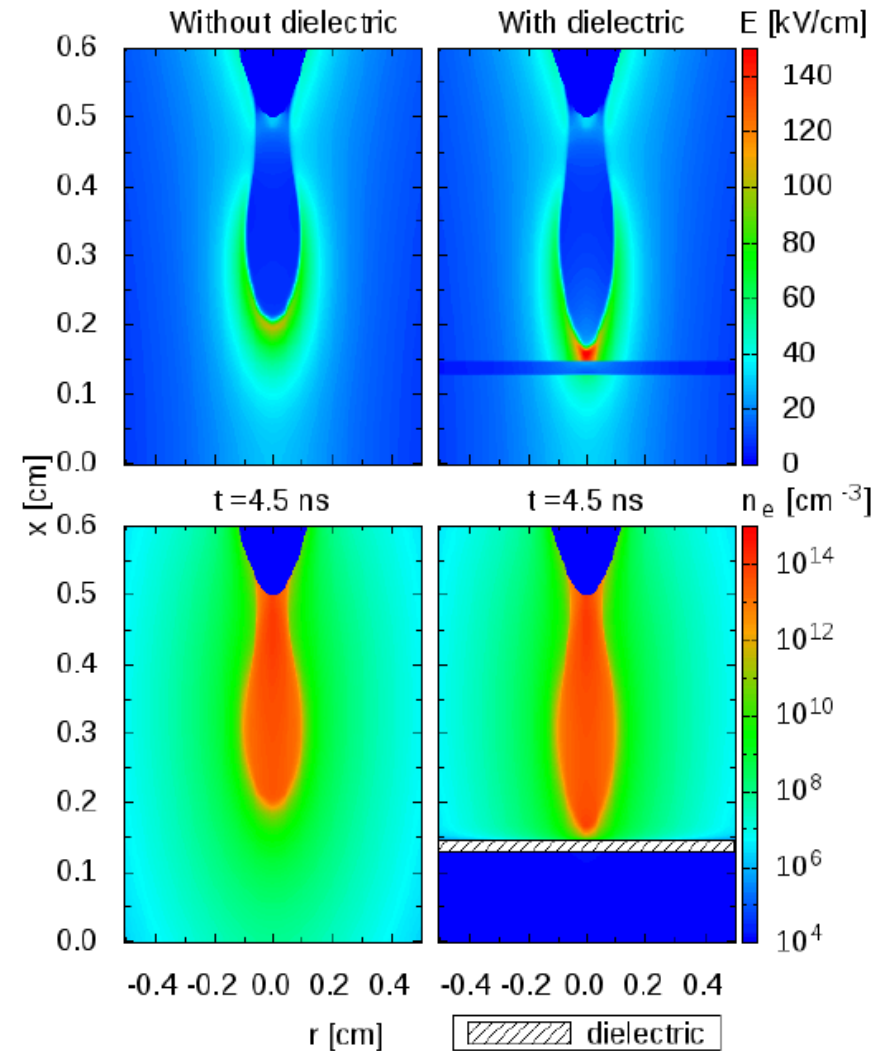
- Ignition of a positive discharge and propagation towards the cathode
- With dielectric:
  - Impact on the upper surface of the dielectric plane at  $t_{\text{impact}} = 4.5 \text{ ns}$
  - Increase of  $|E|$  below the dielectric plane
  - Spreading of the first discharge on the upper surface of the dielectric plane
  - Reignition of a secondary discharge close to the bottom surface at  $t_{\text{rei}} = 5.8 \text{ ns}$



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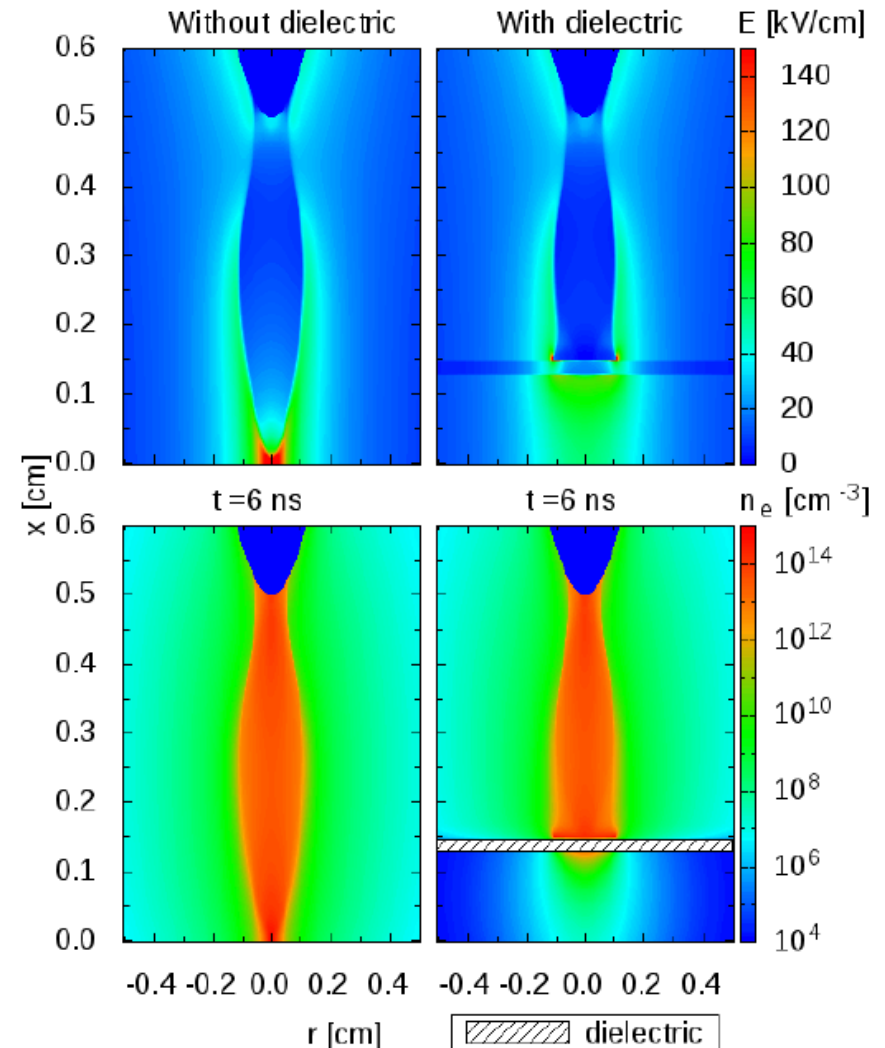
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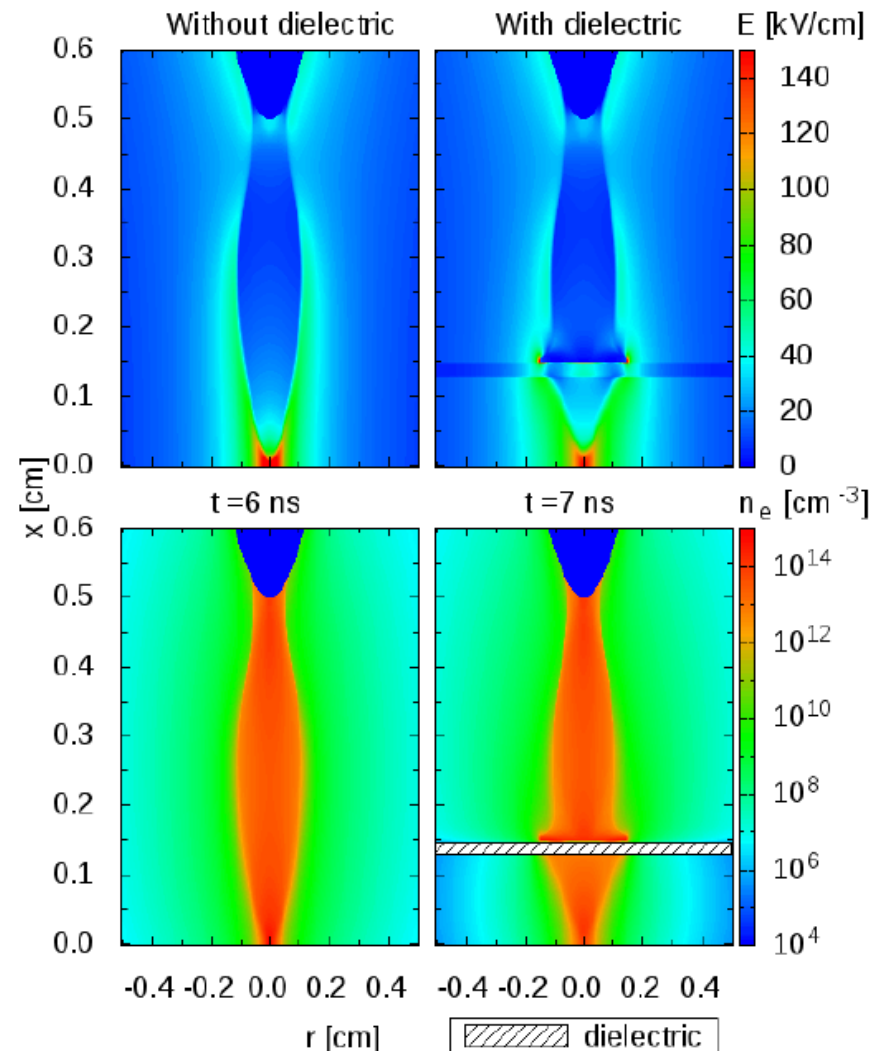


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# Influence of surface charges



Constant applied voltage

$V_{\text{anode}} = +13 \text{ kV}$

- At the time of impact

$$t_{\text{impact}} = 4.5 \text{ ns}$$

$$|\sigma| = 1.10^{-4} \text{ nC.cm}^{-2}$$

$$|Q_{\text{tot}}| \approx 1.10^{-6} \text{ nC}$$

- At the time of reignition

$$t_{\text{reignition}} = 5.8 \text{ ns}$$

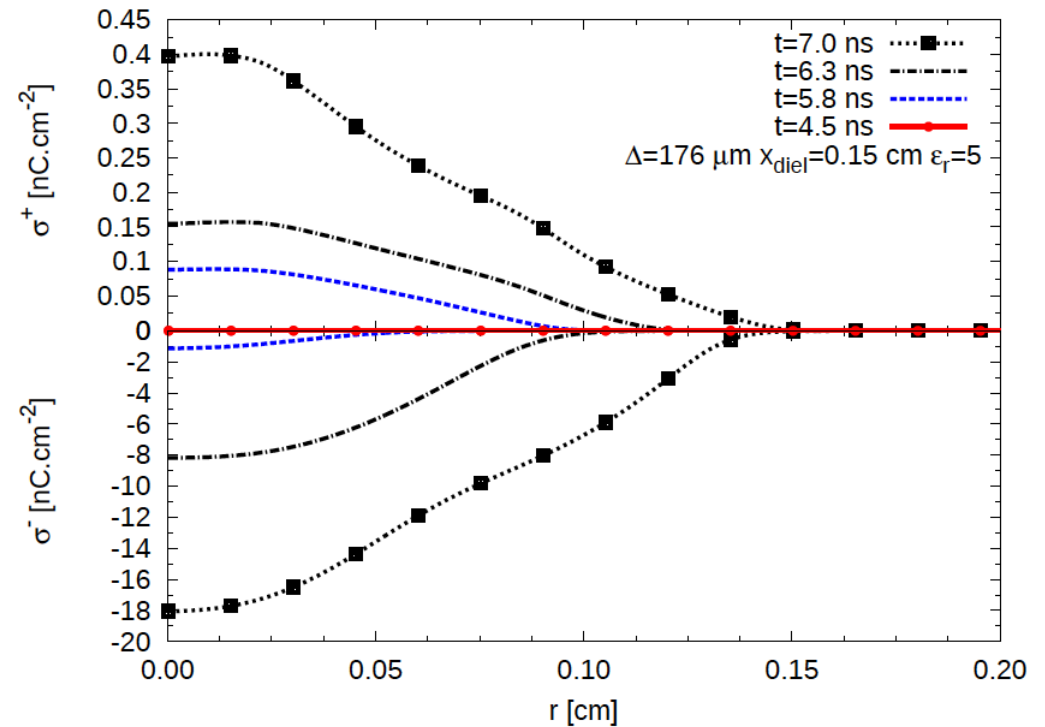
$$|\sigma| = 1 \text{ nC.cm}^{-2}$$

$$|Q_{\text{tot}}| \approx 1.10^{-3} \text{ nC}$$

(much less than DBD case)

- At the time of reignition

$$|E_{\sigma}| \leq 1 \text{ kV.cm}^{-1}$$



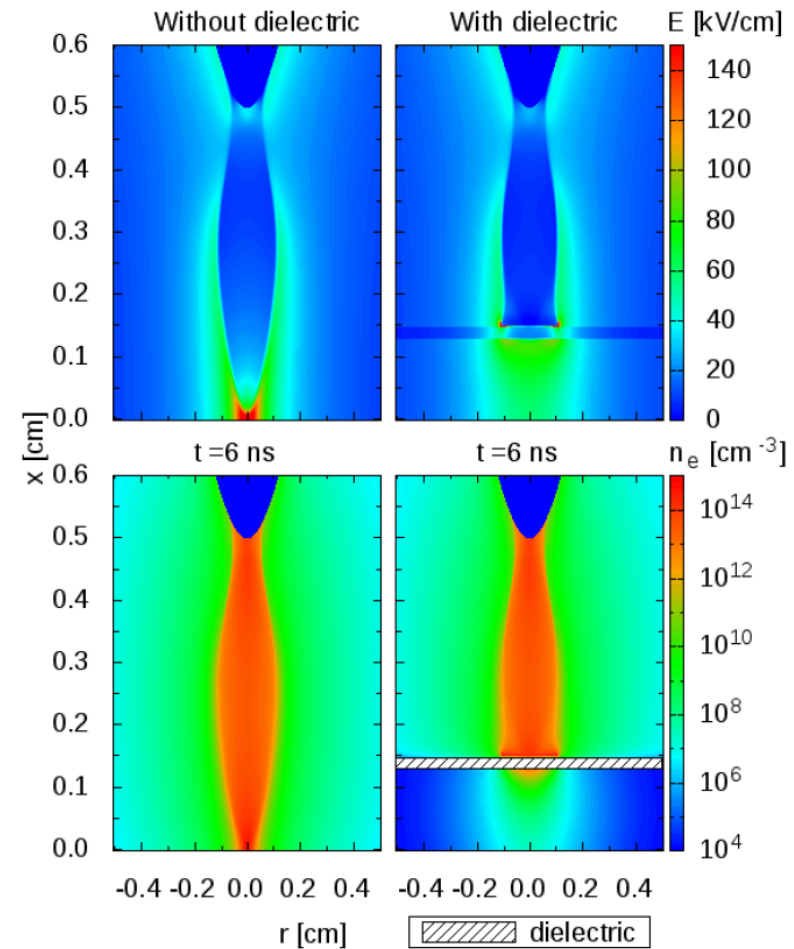
# Influence of surface charges



Constant applied voltage

$$V_{\text{anode}} = +13 \text{ kV}$$

- Reignition:  
High electric field  $|E|$  is required:
  - 1 Surface charges : **Too low to have an influence on reignition**
  - 2 Electric field effect:  
Influence of the first discharge propagation
- Similar results are found in:  
Z. Xiong et al., *Journal of Physics D: Applied Physics* **46**, (2013)





# Influence of the position of the dielectric



Constant applied voltage

$V_{\text{anode}} = +13 \text{ kV}$

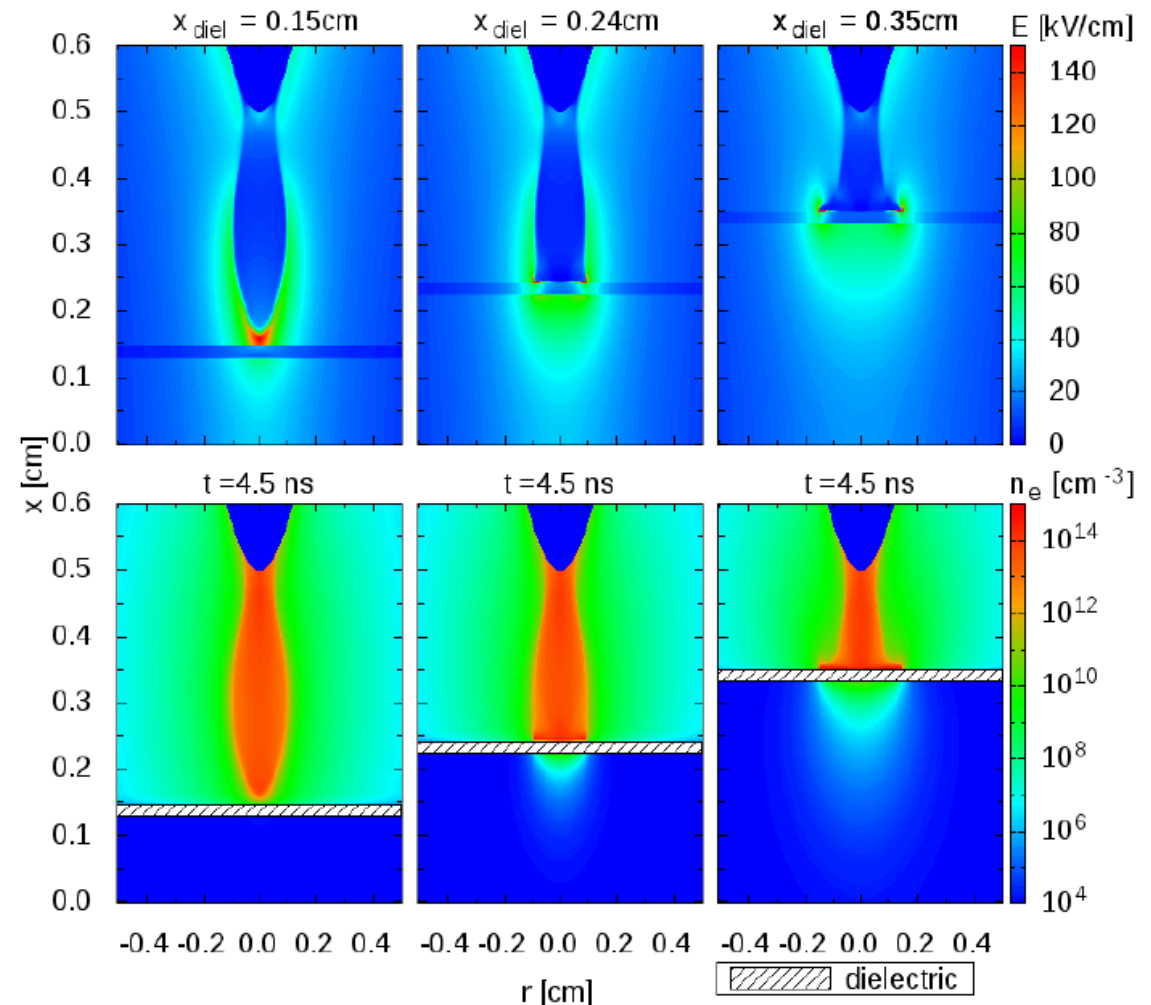
- Impact:

The **closer** the dielectric plane is to the anode:

The **earlier** the impact of the first discharge

For  $x_{\text{diel}} = 0.35 \text{ cm}$ :

- The discharge has more time to: charge the dielectric spread on the dielectric
- At 12.8 ns: No reignition occurred



# Influence of the dielectric



Constant applied voltage

$V_{\text{anode}} = +13 \text{ kV}$

- Impact:

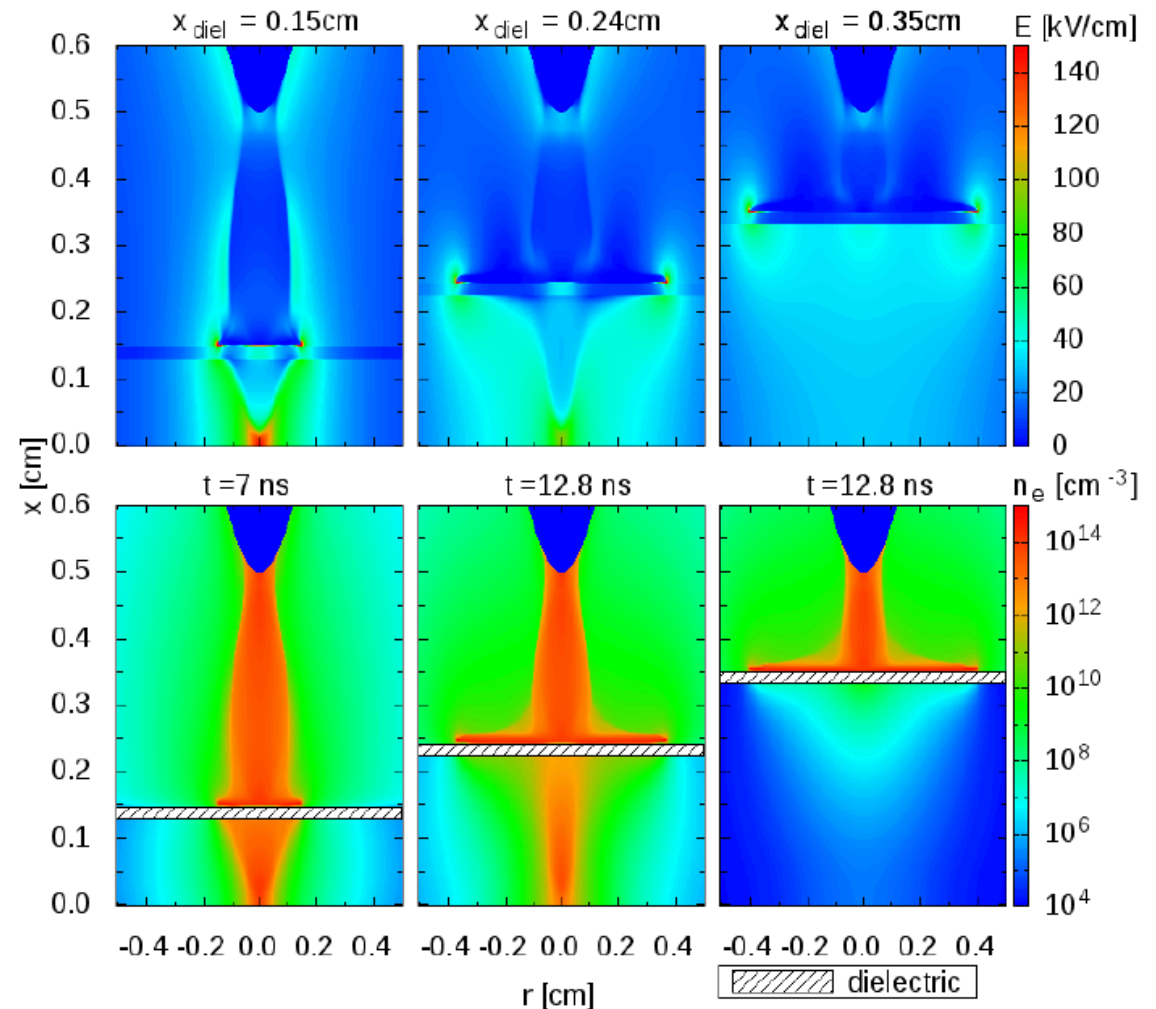
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For  $x_{\text{diel}} = 0.35 \text{ cm}$ :

- The discharge has more time to: charge the dielectric spread on the dielectric

- At 12.8 ns: **No reignition occurred**



# Current challenges for low-temperature plasma simulations



## Numerical studies carried out in parallel to experiments

- Interaction of several discharges (reconnection or merging of discharges)
- Interaction of discharges with surfaces
- Multiscale simulation of the interaction of discharges and reactive flows from mm to several cms and from ns to several ms
- Simulations of plasmas in dense media or in interaction with dense media

## Development of new simulation tools

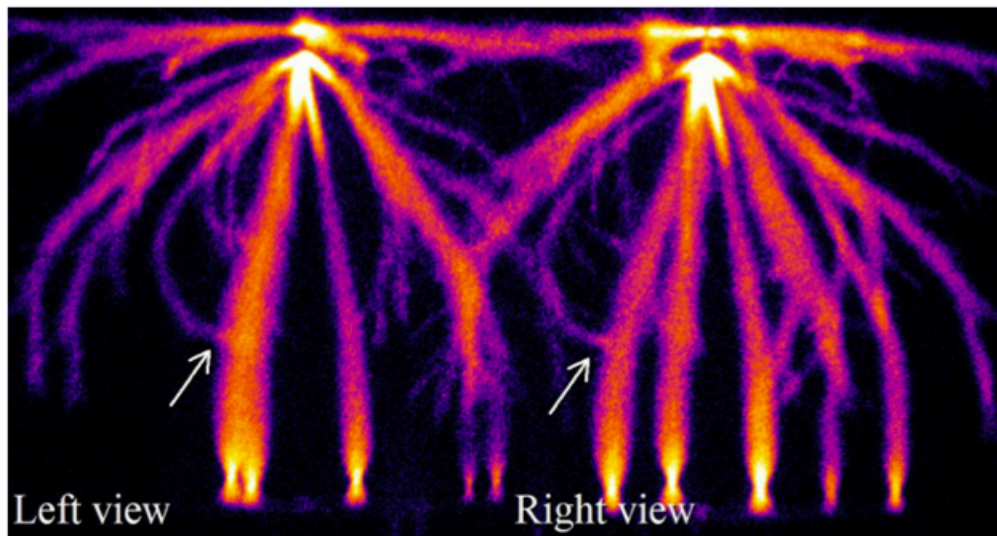
- Higher order fluid models or hybrid models => extension to 3D?
- Time-adaptative and space adaptative multi-resolution discharge codes

# Interaction of discharges

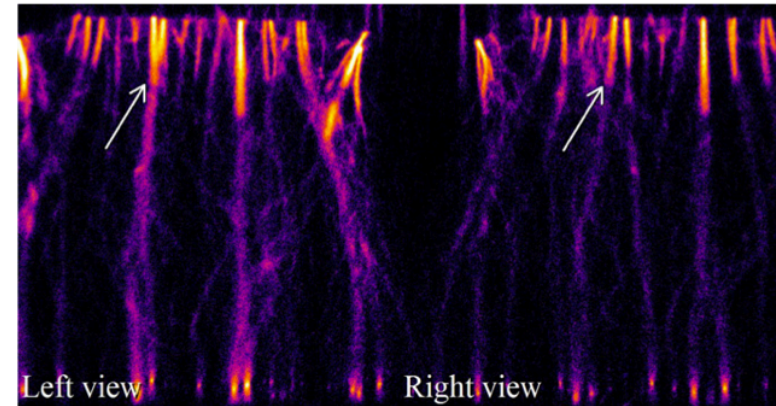


- In nature, streamers appear frequently in trees or bundles
- In plasma reactors for applications, usually, many discharges are generated simultaneously

# Reconnection and merging of discharges



**Figure 5.** A stereo image of a streamer reconnection in single tip anode geometry (*a*). The two views of the single tip event overlap a bit in the middle of the figure. The most striking reconnection location is marked with an arrow in both views. Experimental settings: gas fill: 1000 mbar ambient air;  $V_{\max} = 52$  kV;  $t_{V10\%} = 87$  ns;  $t_{\text{rise}} = 24$  ns;  $t_{\text{start}} = 52$  ns;  $t_{\text{gate}} = 50$  ns. The voltage curve in this experiment is very similar to the one shown in figure 4. As can be seen from the timing parameters of this experiment, the complete image is shot before the voltage pulse reached its maximum.



**Figure 10.** Stereo image of a wire-plate discharge. A possible merging location in the left-hand view is indicated with an arrow. However, the right-hand view clearly shows that in reality no merging occurs. Experimental settings: gas fill: 1000 mbar ambient air;  $V_{\max} = 45$  kV;  $t_{V10\%} = 15$  ns;  $t_{\text{rise}} = 22$  ns;  $t_{\text{start}} = 0$  ns;  $t_{\text{gate}} = 1000$  ns.

# Simulation of streamer merging: 3D cylindrical configuration

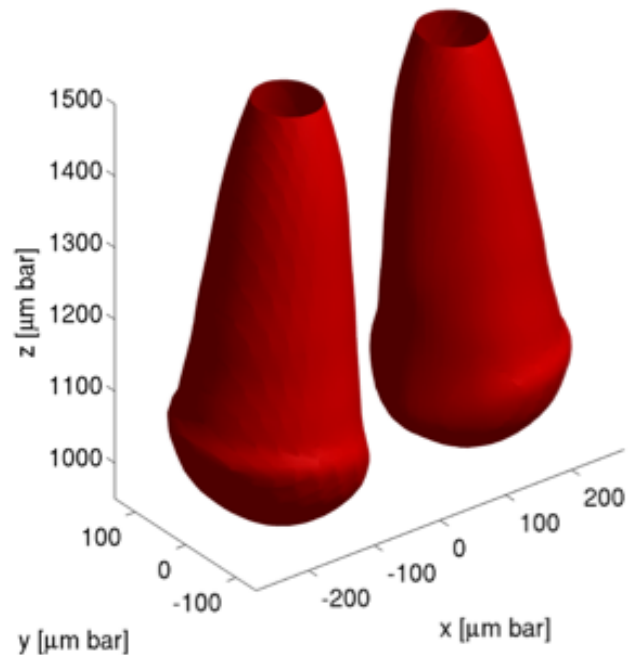
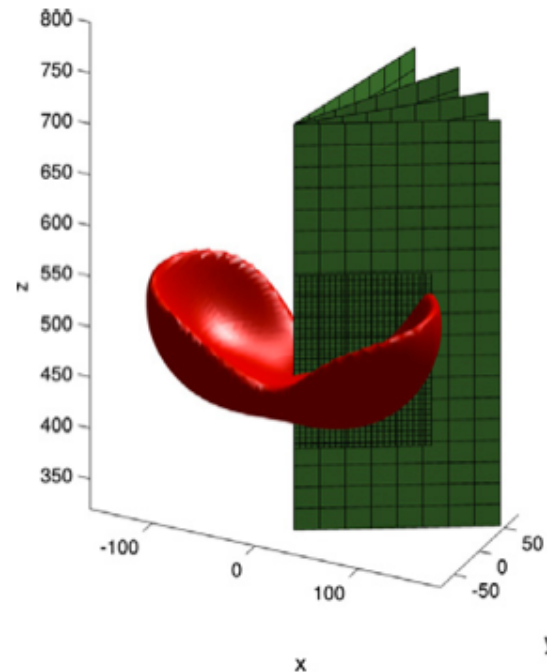


FIG. 1 (color online). Two negative streamers in nitrogen at atmospheric pressure advancing downwards and repelling each other; shown are surfaces of constant electron density in an advanced state of evolution within a constant background field.

Electrostatic repulsion between charges of same polarity in discharge heads  
⇒ Bending of streamer channels outwards



However, for gases as in air, photoionization between the two heads can counteract the electrostatic repulsion between them  
⇒ Merging of streamers



# Simulation of streamer merging

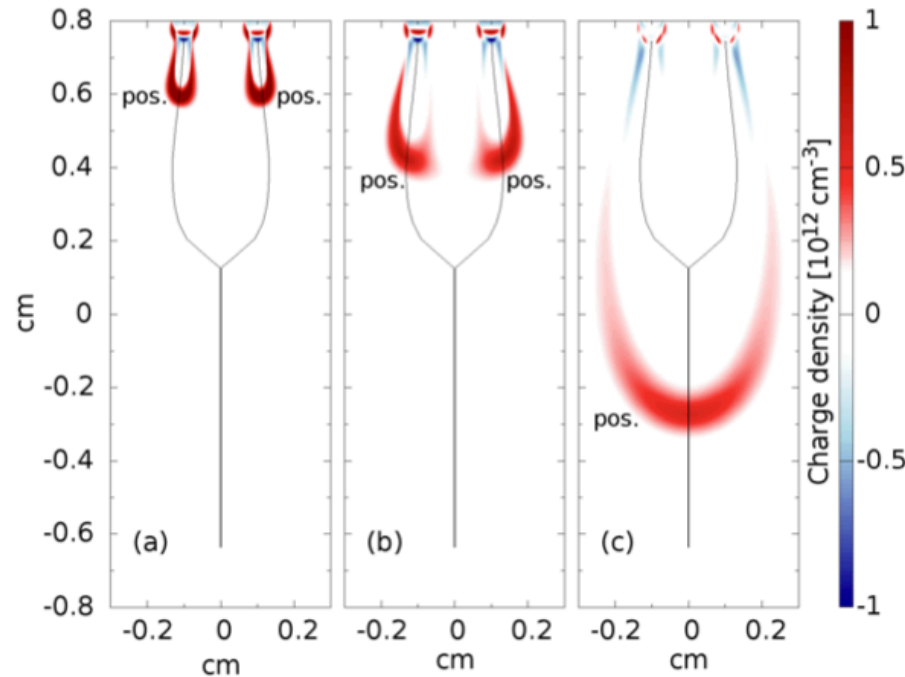


FIG. 1. (color online) Time evolution of the net charge density for two positive streamers at ground pressure ( $N/N_0 = 1.0$ ) for an applied electric field  $E_a = 1.5E_{bd}$  and two Gaussian seeds with  $y_0 = 0.2 N_0/N$  cm,  $n_{\max} = 10^{13} N^2/N_0^2$  cm $^{-3}$ , and  $\sigma = 0.02N_0/N$  cm. (a)  $t = 5.0$  ns: well-developed streamers repulsing each other, (b)  $t = t_{tr} = 6.3$  ns: transition between repulsion and merging, (c)  $t = 8.0$  ns: propagation of a single discharge. Black solid line: trajectory of the maximum electric field.

- For 2 positive or 2 negative streamers in air, merging is obtained when the mutual separation of both streamers are smaller or comparable to the longest characteristic absorption length of photoionization in air
- Based on 2D simulations:  
Determination of a quantitative criterion for streamer merging =  $f(\text{streamer diameter, distance between both filaments})$

# Simulation of streamer merging



movie



# Current challenges for low-temperature plasma simulations



## Numerical studies carried out in parallel to experiments

- Interaction of several discharges (reconnection or merging of discharges)
- Interaction of discharges with surfaces
- Multiscale simulation of the interaction of discharges and reactive flows from mm to several cms and from ns to several ms
- Simulations of plasmas in dense media or in interaction with dense media

## Development of new simulation tools

- Higher order fluid models or hybrid models => extension to 3D?
- Time-adaptative and space adaptative multi-resolution discharge codes



**Thank you for your attention**